Le-Math

Learning mathematics through new communication factors

MATHFactor Guidelines

for Teachers and Students

Lifelong Learning Programme

EN

Lifelong
Learning
Programme

Le-MATH

**Learning mathematics through
new communication factors
2012-2014
www.le-math.eu**

526315-LLP-2012-CY-COMENIUS-CMP

Guidelines for MATHFactor Method

**Teaching and learning mathematics through
Mathematics communication activities**

Guidelines for Teachers and Students

Contribution for the preparation of these Guidelines

The Guidelines are the outcome of the collaborative work of all the Partners for the development of the Le-Math Project, namely the following:

*** Coordinating Organization:**

Cyprus Mathematical Society (CY - Gr. Makrides, A. Philippou, C. Papayiannis, A. Charalambous, S. Christodoulou) along with 12 partners from Cyprus, Greece, Bulgaria, Romania, Austria, Sweden, France, Spain, Czech Republic, Belgium and Hungary.

Partner organizations:

Thales Foundation Cyprus (CY - A. Skotinos, P. Kenderov, E. Christou, L. Zeniou-Papa, C. Christou), **Charles University in Prague - Faculty of Education** (CZ - J. Novotna, A. Jancarik, K. Jancarikova, J. Machalikova), **Loidl-Art** (AT - H. Loidl), **VUZF University** (BG - S. Grozdev), **"CALISTRAT HOGAS" National College Piatra-Neamt** (RO - N. Circu, L-M. Filimon), **Lyckeskolan** (SE - M. Lydell Manfjard), **LEOLAB** (ES - M. Munoz, B. Dieste, E. Cid), **Junior Mathematical Society Miskolc** (HU - P. Kortesi), **European Office of Cyprus** (BE/CY - R. Strevinioti, D. Tsikoudi, C. Katsalis), **Collège Saint-Charles**, **Guipavas, France** (FR - K. Treguer, E. Gueguen, E. Darees, C. Kervernic), **Institute of Communication and Computer Systems**, **National Technical University of Athens** (GR - K. Karpouzis, A. Christodoulou), **Com2go Ltd** (CY - G. Economides, N. Nirou, V. Cheminkov).

Contact the coordinator:
Gr. Makrides at **makrides.g@ucy.ac.cy, thales@usa.net**
T. +35799641843
www.le-math.eu, www.cms.org.cy, www.thalescyprus.com

TABLE OF CONTENTS

GENERAL REMARKS

Section G1. Introduction

The European Project Le-MATH developed, among other, a new method for the learning of mathematics based on mathematics communication, called the MATHFactor method. This method is expected to help, motivate and teach mathematics to pupils of ages 9-18, while using their communication skills.

These Guidelines are designed to be mainly used by teachers and pupils of ages 9-18. However, pupils could benefit by reading it themselves. The Guidelines are part of the MATHFactor Guide Book of the Le-MATH project, which contains besides these guidelines the Manual of Scripts for MATHFactor and a set of sample videos of MATHFactor, presented by pupils from two different age groups 9-13 and 14-18.

The Manual of Scripts for MATHFactor is useful for teachers and pupils who would like to use them for developing a MATHFactor mathematics communication for the knowledge and promotion of mathematics. The manual is expected to be one of the materials used during the Le-MATH training course, developed by the project Le-MATH. In this book, under Annex 1, one can find the analysis of the scripts published in the Manual of Scripts for MATHFactor. The study is available only in the English language while the guidelines are being published in nine European languages such as Czech, Bulgarian, French, German, Greek, Hungarian, Romanian, Spanish, and Sweden.

The project Le-MATH is funded by the European Commission through the programme action Comenius MP from November 2012 to October 2014.

Section G2. What is the aim of MATHFactor?

Unfortunately, many pupils as well as parents consider mathematics to be a difficult and boring subject. Instead of studying mathematics (and other subjects) many pupils prefer to spend most of their time watching TV, playing electronic games or exchanging messages, photos, videos and playing games on their mobile phones. One way to attract pupils back to the "playing field" of education is to use similar tools (weapons) to compete with their "opponents". That is to communicate the learning of mathematics using non-traditional methods such as games, through theatre or using competitions similar to the well-known X-Factor.

Many pupils claim that mathematics is too abstract and therefore non-approachable. This project uses an entirely different and new approach by inviting teachers and pupils to apply new communication methods of learning mathematics, which are fun, enjoyable and functional at the same time. The pupils can "play and learn".

The aim of the MATHFactor is to encourage students to stimulate the imagination of the public and express mathematical ideas using theatrical skills to a non-specialist audience.

More precisely these Guidelines focus on the development of the methodology in teaching and learning mathematics through the creation of a tool called MATHFactor and which is to provide the basics for "Teaching and learning mathematics through communication activities".

In the proposal it is specified that this method (MATHFactor) intends to bring communication activities, that are widely spread today, to the classroom (e.g. social media, TV-shows and games) and use them as methods and tools for improving learning, to increase the interest of pupils in order to become more active and creative and to actively involve them in the learning process. It is proposed to develop this new method as a teaching tool for teachers and as a learning tool for pupils, where pupils will be encouraged to communicate mathematics with a new approach.

Teachers will be able to teach and train their pupils on how to explain a mathematical theorem, method, or a mathematical application in a way that

can be understood, appreciated and enjoyed by non-experts. It is known from previous research that learning through reading is absorbed and sustained only at 10%, but experimental learning and learning through explaining mathematics could help students to absorb and sustain knowledge up to 90%.

These Guidelines are providing the framework that will enhance teachers' skills. Through this tool the pupils will be encouraged to communicate ideas of mathematics, to comprehend various concepts, processes and ideas that have mathematical context, to indulge in the philosophy and history of mathematics, to reflect on the characteristics of the pioneers in the area and to develop moral, aesthetic values that are inherent in the subject.

More specifically, pupils (and certainly teachers) are expected to explain/ present/communicate to others:

- A mathematical concept
- A mathematical theorem
- A mathematical method
- A mathematics application in a way that can be understood, appreciated and enjoyed by non-experts.

Through these Guidelines the reader is expected to become acquainted with some aspects of "the state of the Art" in the area and to be able to answer:

- What are the objectives of Mathematics and how can the MATHFactor approach help (or how can the MATHFactor approach be of value)?
- What are some of the basic aspects of the theoretical background concerning the exploitation of MATHFactor as a learning approach?
- What are some of the models/approaches/examples when using MATHFactor activities as supporting means in learning/teaching?
- What could be the practice in integrating MATHFactor activities in teaching?

Furthermore, these Guidelines could be of value to teachers in **designing scenarios/presentations for teaching/learning**. Among these, we would expect the development/acquirement of competences in issues like:

- The development of a scenario – by the teacher or the student – for a presentation based on mathematical ideas, aiming to motivate and improve communication skills in the context of the mathematical education of pupils.
- The development or the adaptation – by the teacher or the student – of a scenario for a presentation based on an existing text or story in the area

of mathematics history, concepts and pioneers; aiming at motivating, facilitating comprehension, reflection and improving skills in the context of the mathematical education of pupils.

- The development of a presentation, by the student, using a scenario that will help in explaining a mathematical concept, process or another idea to his/her colleague students or to other non-experts.
- The participation of students in presentations and communication activities as learning/comprehension means for a mathematical idea, process, concept or related act to the educational values of the topic.

Through these Guidelines, it is expected that teachers will develop competencies for implementing/applying MATHFactor activities/scenarios for teaching/ learning. Among these it is expected to provide opportunities for the discussion of issues like:

- The recognition and use (in the context of the usual math class or in the context of other activities, curricular or extracurricular) of MATHFactor activities/scenarios/presentations aiming at motivating students and improving various mathematical skills in the context of the mathematical education of pupils.
- The identification and use of MATHFactor activities/scenarios in the area of mathematics history, concepts and pioneers, aiming at introducing the students to new topics or in enriching their mathematical experience.
- The identification and use of MATHFactor activities/scenarios/presentations in the area of mathematics, aiming at helping/explaining or in the understanding of a mathematical concept, process or other ideas to students.

Finally, the Guidelines are expected to provide the teachers with some information concerning **the organizing/participating in festivals/competitions with MATHFactor activities related to mathematics.**

PART A - Methodology

Section A1. Why Mathematics Communication - Putting new theories into new practices

Mathematical communication is an essential process for learning mathematics, because through communication students reflect upon, clarify and expand their ideas and understanding of mathematical relationships and mathematical arguments. (Ontario Ministry of Education, 2005)

Dialogue is part of the mathematical communication that occurs in the classroom. Effective communication happens when students articulate their own ideas and seriously consider their peers' mathematical perspectives as a way to construct mathematical understandings. Encouraging students to construct their own mathematical understanding through communication is an effective way to teach mathematics, especially since the role of the teacher is transformed from being a transmitter of knowledge to one who presents worthwhile and engaging mathematical tasks. *Professional Standards for Teaching Mathematics (NCTM 2000)* identifies communication, with discourse as an essential component, as one of the 10 Standards for teaching mathematics.

"Because mathematics is so often conveyed in symbols, oral and written, communication about mathematical ideas is not always recognized as an important part of mathematics education. Students do not necessarily talk about mathematics naturally; teachers need to help them learn how to do so". (Cobb, Wood, & Yackel, 1994).

"Oral Communication includes talking, listening, questioning, explaining, defining, discussing, describing, justifying, and defending. When students participate in these actions in an active, focused, and purposeful way, they are furthering their understanding of mathematics".
(Ontario Ministry of Education, 2006, p. 66).

Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to others'

explanations gives students opportunities to develop their own understandings (NCTM, 2000, p. 59).

Students need opportunities to test their ideas on the basis of shared knowledge in the mathematical community of the classroom to see whether they can be understood and if they are sufficiently convincing. When such ideas are worked out in public, students can profit from being part of the discussion, and the teacher can monitor their learning (Lampert, 1990).

Categories of Mathematical Communication:

- Expression and organization of ideas and mathematical thinking using oral, visual, and written forms
- Communication for different audiences and purposes
- Use of conventions, vocabulary and terminology of the discipline (in oral, visual, and written forms
  (Ontario Ministry of Education, 2005, p. 23)

These categories listed below in more detail:

- **Expression and organization of ideas and mathematical thinking** (e.g. clarity of expression, logical organization):
    - Using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials).
    - Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson (Silver, Kilpatrick, and Schlesinger 1990).

- **Communication for different audiences and purposes**
    - The teacher should encourage students to express their mathematics ideas by using a combination of oral, visual and written forms for this justification. Students should be able to express their mathematic ideas to different audiences such as teachers, peers, family and the mathematical community.

- To the teacher: Students should justify their solution when they solve a problem or a mathematical task. Some ways that this can be done is through homework, or through a test. In either case, a full explanation to the student is not available unless the teacher engages in one to one communication with the student.
- To the peers: Students should be encouraged to express such ideas or justifications to their peers. This can be done by having the students present the mathematical issues to the whole classroom or to a team of classmates. Another way that this can be achieved is by having a mathematics debate in the classroom. Also by encouraging students to do a mathematics project, where they will need to interact and persuade one another in order to come up with the final product.
- Students should also try to question and discuss with others, any mathematical concepts that are not very clear to them, in order to better comprehend that concept. They should also try to understand someone else's thinking and to examine mathematical methods different than their own. In other words, they should learn to be critical thinkers.
- To the family or to the community: Students can help or get help from other members of the family. Furthermore they can use mathematical ideas to solve an everyday problem that arises in the family, in the neighborhood or the community.
- To the mathematics community: by taking part in a mathematics conference or a mathematics competition.

  As students practice communication, they should improve their clarity and coherence in their communication. They should also acquire and recognize conventionally mathematical styles of dialogue and argument. As they progress, their arguments should become more complete and should draw directly from the shared knowledge of the classroom. Over time, students should become more aware of, and responsive to their audience, as they explain their ideas in mathematics class. They should learn to be aware of whether they are convincing and whether others can understand them. As students mature, their communication should reflect an increasing array of ways to justify their procedures and results. In the lower grades, providing empirical evidence or a few examples may be enough. After they become more familiar with this process, students are expected to be able to produce short deductive chains of reasoning, based on previously

accepted facts. In the middle grades and high school, explanations should become more detailed and the students should increasingly state in their supporting arguments the mathematical properties they used. (Professional Standards for Teaching Mathematics, NCTM 1991).

- **Use of conventions, vocabulary and terminology of the discipline in oral, visual, and written forms**.
  Students tend to use their everyday language in order to express their mathematical ideas. The teacher should try to help them use a precise mathematical language using the correct terminology and definitions.

  The teacher should be able to make a connection between mathematical language and everyday language, in order to make the students understand that mathematical concepts may derive from everyday activities. Words such as limit, groups, circle, and straight line are words that apply in both everyday and mathematical language. Therefore, it should be made clear to the student what are the similarities and differences between the two languages, so that they will be able to make the distinction between the two. Many times, when students explain something in their own words it gives them a feeling of accomplishment, and this should be encouraged. At the same time, the teacher should make the appropriate corrections. For example, if a student uses the words “normal triangle” instead of “regular triangle”, the teacher should give emphasis to the part of the student’s explanation that is correct, but at the same time he should note the proper terminology.

  Beginning in the middle grades, students should be able to understand the role of mathematical definitions and should use them in their mathematical work. Doing so should become pervasive in high school. However, it is important to avoid a premature rush to impose formal mathematical language. Students need to develop an appreciation for precise definitions and for the communicative power of conventional mathematical terms by first expressing them in their own words. Allowing students to grapple with their ideas and to develop their own informal means of expressing them, can be an effective way to foster engagement and ownership (NCTM).

  As students progress through the grades, their method of communicating mathematics should become more complex and more abstract. Students’ repertoire of tools and ways of communicating, as well as the mathematical

reasoning that supports their communication, should become increasingly sophisticated. Support for students is vital, especially for those whose primary language is not the same with the one used in classroom. These students may need some additional support in order to benefit from communication-rich mathematics classes, but they can participate fully if classroom activities are appropriately structured (Silver, Smith, and Nelson 1995).

The Five Practices Model

The teacher's role is to:

- anticipate student responses to challenging mathematical tasks;
- monitor students' work and their engagement with the tasks;
- select particular students to present their mathematical work;
- sequence the students' responses that will be displayed in specific order; and
- connect different students' responses and connect the responses to key mathematical ideas.

(Smith, M. S., E. K. Hughes, R. A. Engle & M. K. Stein 2009)

Students should become more skilled in speaking and in convincing or question their peers. The conversations in the classroom should focus on making mathematical ideas simple and logical. It should also focus on using mathematical ideas on solving a problem effectively through mathematical modeling.

A student should be able to present mathematical ideas to other students and also be able to listen to other students' ideas.

They should not be afraid of joining group discussions in order to clarify, question, and extend conjectures. This involves speaking in the classroom in order to convince or question peers.

Even though discourse is not a goal in mathematical teaching, it is certainly a mean for achieving greater mathematical understanding and for spreading mathematical ideas among students.

Even more effort should be applied so that the student would be able to present his mathematical ideas in front of strangers or in front of an audience.

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely (NCTM 2000).

How does communication apply to MATHFactor?

In order for a student to succeed in MATHFactor he must reach a point where he can take a mathematical concept and turn it into a problem or a simple story, and then organize all the facts of that concept in a logical order and consolidate mathematical thinking through oral communication. When a student can do all that, then we can say that he understood the mathematical concept.

Also, he must be able to express his story or his problem with clarity by using different communication means, oral communication and/or body language. In addition, he can use mathematical constructions, graphic representation or any other material that will help him present his idea in a correct mathematical way. Through the MATHFactor competition, students have the opportunity to present a mathematical idea or mathematical concept, in a way that will be understood by an audience with no mathematical background.

During their presentation, they should be able to recognize if their audience follows and understands them, and if their mathematical solution is correct.

The mathematics language used by the students should be precise with accurate definitions, terminology and correct use of graphs and symbols. If the term is one that the audience will not be able to understand the student will have to find a way to explain it in simple, understandable terms.

Section A2. Mathematics Communication Factors for learning mathematics in the school environment and beyond.

Setting the goals and objectives

Communication is the combination of methods of transferring information (content, message, signal) between two parts, the sender and the receiver, using a variety of methods (written words, nonverbal gestures, spoken words). We use it also to establish and modify relationships. In some cases, the contact is considered restricted to verbal communication, and the other, non-verbal communication aspects are regarded as part of meta-communication, which can enforce or weaken the affectivity of communication.

We will use the terms of verbal and non-verbal communication. The communication of mathematics needs particular analysis, because beside the general communication factors there are also other factors to consider that are more specific to the learning of mathematics.

First of all, any mathematical communication must be preceded by a deep understanding of the problem, and the mathematics behind it. This is a special period, when you make up your plan and choose the right communication strategy.

Sometimes, like when you perform a MATHeatre or MATHFactor piece, you may have plenty of time to prepare it. In many other cases though, like when you discuss with your classmates, or when you answer the questions of your teacher during a lesson, the preliminary phase could be very short. In order to be able to acquire the right communication skills, you need first to understand the basics of communication skills.

Understanding the Basics of Communication Skills

Have the courage to say your point of view.
Be aware of the fact that you can make useful contributions to the conversation. Take the time to clarify in your mind your opinion, so that you can adequately explain it to others. Pupils often hesitate to speak in the classroom, because

they lack self-confidence and they fear that they might give a wrong answer. You should keep in mind that something that is important to one person may not be to another. It is upon your presentation to show why it could be of interest. You can start by pointing out why the given subject is interesting to you.

Try to engage your audience - Capture their attention - Make and keep eye contact.

You should remember that whether you are speaking or listening, if you keep eye contact you can make a conversation much more interactive. In the case of a dialogue between two people, it helps to consciously to look into the listener's eyes. Eye contact conveys interest and encourages your partner to be interested in you in return. If you have a larger audience, try to fix on 3-4 people, but from time to time look around the room, like you try to find somebody you know.

When recorded by a camera, try not to stare at the lens, look close to the camera, feel its presence, pay attention to it, say a few words directly looking at the camera, but avoid creating the feeling that you communicate only through the camera. You need to turn to other spectators, the cameraman, one of your colleagues; you even can address somebody in the room to create the right conditions for your performance.

Use gestures.

Use gestures, both, with your hands and face, even with your whole body posture. Use smaller gestures for individuals and small groups. The gestures should get more abundant as the audience increases in size. Be aware that gestures have deep cultural roots; you need to know the local particularities. Some gestures like nodding, have different meaning in some countries. For instance, in France and Hungary nodding means yes and in Bulgaria it means no.

Do not send mixed messages.

Make sure that your verbal and non-verbal messages, gestures, facial expressions, posture, body language and tone match each other. Quarreling with someone while smiling sends a mixed message and therefore it is ineffective. If you deliver a positive message, make sure your words, facial expressions, and tone match the message.

Be aware – Your body is supporting what you are saying.
Body language can say much more than words. An open stance with arms relaxed at your sides shows to the people around you that you are approachable and open to hear what they have to say.

On the other hand, crossed arms and hunched shoulders suggest disinterest in conversation or unwillingness to communicate. Often, communication can be stopped before it starts by body language that tells people you do not want to talk. Appropriate posture and an approachable position can make even difficult conversations flow more smoothly.

Manifest constructive attitudes and beliefs.
The attitudes you bring to communication will have a tremendous impact on the way you compose yourself and interact with others. Choose to be honest, patient, optimistic, sincere, respectful and accepting. Be sensitive to other people's feelings and believe in others' competence.

Develop effective listening skills.
Not only you should be able to speak effectively, you must also be able to listen to the other person's words and engage in conversation according to their thoughts. Avoid the impulse to listen only the end of their sentences and avoid interrupting them while they are talking.

Use Your Own Words

Enunciate your words.
Speak clearly and don't mumble. If people are always asking you to repeat yourself, try to do a better job in articulating yourself.

Pronounce your words correctly.
People will judge your competency through your vocabulary. If you are unsure of how to pronounce a word, do not use it.

Use the right words.
If you are not sure of the meaning of the word, do not use it. Start a daily habit of learning a new word each day and try to use it when it is possible in your daily conversations.

Slow down your speech.
People will perceive you as nervous and unsure of yourself if you talk fast. However, be careful not to slow down to the point where people begin to finish your sentences.

Use Your Voice

Develop your voice – A high-pitched or whiny voice is not perceived to be one of authority.
In fact, a high-pitched and soft voice can make people to not take you seriously. Begin doing exercises to lower the tone of your voice. Try singing your favorite songs, but do it an octave lower. Practice this, and after a period your voice will begin to deepen.

Animate your voice.
Avoid a monotone voice and use dynamics. Your pitch should raise and lower periodically. Radio DJ's are usually a good example of this.

Use appropriate volume.
Use a volume that is appropriate for the setting and content. Speak more softly when you are having a more intimate conversation. Speak louder when you are speaking to larger groups or across larger spaces.

It is good to keep in mind the following tips in order to be efficient and to develop good communication skills:

- Try to speak fluently and try to make sure people can hear you when you speak.
- Get feedback from your receiver to ensure that you were properly understood during your conversation.
- A good speaker is also a good listener.
- Make sure you are using proper grammar.
- Have confidence when presenting, it does not matter what other people think.
- Do not interrupt or talk over the other person--it breaks the flow of conversation. Timing is important.
- Do not over-praise yourself in front of your audience.

Section A3. Motivation

Motivation and MATHFactor

Motivation is necessary for high results in learning and for acquiring better skills in the educational process. Without motivation, the learning ability decreases correspondingly. Learning starts with birth and continues during our whole life. It is inherent for people, no matter whether it runs or not in a formal environment. Thus, students need special stimuli for active, purposeful and constant efforts in learning. In a broader sense, motivation is what influences creativity, supports the ability to choose and the direction as a whole in human behavior. It also pushes students to explore, request, transform and to use knowledge. It is connected with the desire for participation in the educational process, but also touches the reasons that lie in the base of active participation in various activities. Although trainees could be motivativated for the fulfillment of a given task, the reasons for their motivation could be different. In other words, a motivated trainee undertakes a given activity because of the activity itself or because of the pleasure it gives him.

People express their skills in different ways. Similarly, they might receive inspiration and be stimulated by different things. The phenomena that help them mastering and acquiring skills could be of quite diverse nature. The MATHFactor aspires to do exactly that—to motivate and inspire students. The pedagogical effectiveness of this method is based on the action approach in education and on teaching principles for problem solving. Its realization leads to the development of internal motivation for learning; development of constructive critical thinking; forming of basic competences, i.e. skills for problem discovery, for purpose

settlement, for planning of actions but also skills for self-analysis and reflection, comparison, analysis, synthesis, prognosticating, independent search, keeping and practical application of the investigated information, presentation in the process of independent activity and its result, initiative, communication and tolerance. With the MATHFactor method, students learn self-organization.

The MATHFactor method is a variation of the practical realization of full integrated education, whose main elements are: purpose, action, unity. Its objective is to strike balance between the knowledge and the practical skills of each student, thus finding opportunities for useful realization of integrated connections among natural foreign language knowledge, abilities and presentation skills. The method directs students to a non-traditional study of chosen problems and requires the creation of concrete educational products, which reflect fully the system of knowledge, skills, relations and the personality's qualities. Working with MATHFactor students self-check themselves, self-control the presentation course, enhance their knowledge in technology, and become conscious of the knowledge significance for the success of their activities. In this way, suitable conditions are elaborated for the acceleration of the personality growth. By synthesizing student ideas and efforts, the finished product generates satisfaction. The positive psychological effect is of extraordinary significance and it creates a fruitful experience for the students.

An important peculiarity of the MATHFactor based teaching methodology is that the verbal activity is dominating. However, it does not overshadow learning by action. Although the differences between the roles of teachers and students are not changed, the leading role of the teacher is not more highly emphasized.

Teachers stand behind and understand their functions only by general planning, general activities and general assessment. A large variety of problems gives possibilities to students to participate in work in numerous ways. They could choose their performances in relation to their previous experience, ambitions and preferences. With MATHFactor students are challenged to look for solutions of real problems, to act actively for the creation of motives for the solutions, to express feelings of responsibility, to construct reasonable answers. The task of the teacher is to ensure necessary tools and to support students in difficulties.

Interesting moments from the history of mathematics, from ancient times and

contemporary applications excite student phantasy and increase the interest of young people. Interest is a complex psychological phenomenon. It comprises consciousness, will and emotions. Interest is a selective purpose of people, of their attention and thought. This goal is characterized by a constant striving to acquire knowledge, to study and elaborate a given subject or a given fact. The interests of an individual depend on his background and personal development. They are quite variable and could be classified in several directions:

- **Curiosity** – elementary stage of orientation. It represents the wish for novelties.
- **Entertainment** – striving to penetrate beyond obviousness. Characteristic of this stage is the emotion, like the surprise and the joy of knowledge.
- **Cognitive interest** – this refers to the notion that the knowledge that people produce is driven by their interests.

The cognitive interest relates to the search of character. Under its influence people independently look for answers to various questions. The searching activity of students is realized by will, the emotional enthusiasm and the joy of success. The interest is directed not only to the content of the presented material, but also towards the actions and the problems, connected with the presentation. Developing itself, the cognitive interest becomes a base for a positive attitude to learning. The interest for mathematics is expressed through the effort to master mathematical knowledge and from cultivating the skills in order to acquire such knowledge. It also tries to find the relation between mathematics and other subjects or phenomena, to explain the applications of mathematics in other disciplines and the surrounding reality and additionally, like in the case of MATHFactor, to impart knowledge to the audience.

The cognitive interest is connected with the possibility for expression, which is one of the most important incentives for learning. This possibility could be transformed into a stable feature of the personality and could influence its development strongly. For this reason, the cognitive interest of students should be excited, developed and consolidated in the learning process by the creation of conditions for expression. MATHFactor not only focuses to the process of knowledge, but also to its results. It is connected with the tendency to realize your purpose and with overcoming difficulties. The development of interest is a process, which follows individual and age peculiarities of the cognitive activity. As a result, it can be argued that interest has a positive effect on the individual and that it can be identified not only by the action, but also by the leading cause.

The leading cause in MATHFactor is the individual expression. It positively affects not only the process and the result of an action, but also the running cognitive activity (thinking, imagination, memory, attention). Except for the mental processes, the expression through MATHFactor includes elements of practical actions, which are related to the explanation of respective mathematical facts or assertions. It is the sum of intellectual, volitional and emotional processes. Thus, all the important personality manifestations interact in an original way. The emotional aspects include surprise, expectation of the audience reaction, the feeling of intellectual delight, and the feeling of success.

Interest can be stimulated by a new and unknown educational material that will positively surprise students. A positive emotional attitude towards self-learning is created by giving the right incentives. A higher level of satisfaction can be generated by using the MATHFactor method as a new, more productive form of learning and communication. In some cases, social motives which are connected with the desire for participation in useful activities, might also surface. Skills can be developed and cultivated from presentations, from the evaluation of different points of view, and from multilateral self-evaluation, which can increase a person's feeling of responsibility. The participation in MATHFactor improves the educational activity and the student motivation. Simultaneously, it helps to identify the function of the social motives, which maintains the learning interest when cognitive motives are missing. New features of mental development appear along various types of activities, which determine student attitude of a new type; towards the subject under study, towards the audience, towards the proper activity.

There are different ways to promote motivation. One possible way is the so called "way from the top". It consists in student mastering of what is taught. Possible insufficiency of this way is dangerous for declarative learning and formalism. Another one is the "way from the bottom," which consists in student inclusion to active actions creating reality motives. The MATHFactor falls under the second type. It can transform a negative and indifferent attitude to learning into a positive one – operatively, by responsibility and consciousness. Performance in front of an audience is a positive motivation for individual and independent expression, which includes the learning activity and self-control. It accounts for subject characteristics of the student (age, sex, intellectual development, skills, belonging, self-assessment, interaction with students of the same age etc.).

Each student possesses his/her own characteristic features and respective motivation sphere. The word “motivation” has common roots with words like “motor,” “moment,” and “mobile– words that are demonstrating motion, physical action. In that sense, motivation goes hand in hand with physical action. Motivation is not what students think or feel, but what they do physically. When teachers want to motivate them to achieve results, they have to provoke them with such actions that will bring the desired results. Consequently, it can be argued that motivation is set in motion by emotion.

Section A4. Content - Linking MATHFactor scenario with the math curriculum

MATHFactor is an active learning programme which includes a competition element.

Active Learning

Active learning includes a wide range of teaching techniques that involve the active participation of students in undertaking tasks and an analysis on why and how they do them. It encourages students to:

- Develop critical thinking
- Work their creative skills
- Improve their writing skills
- Understand better themselves and how they learn
- Cooperate and help each other to get better through constructive feedback

Active learning techniques can be applied either inside or outside the classroom, in a formal or non-formal learning process, in indoor or outdoor activities, in teams or personally, and by using modern technological or traditional means. Teachers who engage with these techniques, use more of their time mentoring students and helping them understand their potential and skills, to help them achieve a higher understanding instead of simply citing information to a passive audience. Moreover, teachers in active learning help the students to improve their presentational and expressive skills, by encouraging them to present their work and ideas and seek for feedback from their classmates and friends, in addition to the teachers' own remarks.

Competitions in Education

Running competitions has been a mean of interactive teaching and active learning since ancient times. A particular example was when Baron Pierre de Coubertin was trying to revive the Olympic Games in 1896 in Athens, Greece, Eötvös University in Budapest, Hungary, held the very first national- level math competition in 1894, most likely inspired by his efforts.

This was the start of a series of national math and science competitions that sprung around central Europe and even reached North America, in 1938.

These were the events that gradually led to the first International Mathematics Olympiad, hosted in Romania in 1959. Other sciences shortly followed and got their own Olympiads: Physics in 1967 and Chemistry in 1969, computing in 1989, Biology in 1990 and last but not least, Astronomy joined the Olympiads Club in 1996.

Apart from the Olympiads that have been established as annual meeting points between school communities, nowadays students are receiving more and more invitations to participate to science and math contests. From Google Science Fair to local schools math championships, competitions are regarded as a strong motive in the learning process. For this reason, educators need to ensure that their students take part in events and contests designed for their age, knowledge and potential. This way they can build students' excitement over specific topics and to boost their. On the contrary, taking part in a competition that is not appropriate for the age or the abilities of a student creates negative impressions and feelings, and can potentially discourage them.

MATHFactor as a concept

Since its inception, MATHFactor has a good track record in assisting students to find and develop their talent in communicating mathematics to a wider public. It uses a high-profile international event to attract people that want to share their enthusiasm for mathematics. The model is similar to TV programmes like Pop Idol or X-Factor: you have just 3 minutes to prove yourself to a panel of expert judges, with only the best going forward to the next round.

MATHFactor gives each candidate exactly three minutes to resolve their topic in a simple and fun way.

Props

"Props," coming from the word "property" is about every object that can be used on stage for demonstration purposes or to make a point.

Use of props

For example, if the topic is about mathematics in ancient pottery, ceramic vase would be a suitable prop.

A prop is usually a useful tool, but how much you encourage, discourage or disallow them is a matter of judgment. Although inexperienced presenters can lose themselves with too many props, others can use them to great effect and make for a more visually interesting show.

Remember that no PowerPoint presentations may be used, and writing or drawing on paper is to be discouraged.

Innovation

MATHFactor from its very beginning was designed to be innovative compared to other school competitions. First of all, its aim was to get the students directly involved with mathematics and to act as little mathematicians who are trying to find out how something works. Having your mathematics communication's target audience act as a mathematician and get the most out of it is MATH factor's first innovation.

The students are no more passive recipients of a concept made for them; they are thinking, playing and learning at the same time.

Skills developed

MATHFactor is a program that is intended and constructed in a way to boost the participants' skills or to encourage them to build some new along the way. Ideas development and ideas presenting, comprehension, passing a message, public speech, communication, even problem solving, modeling, and analytical thinking are present in the process. The participants will need them all, in order to express their ideas and to inspire their audience by delivering a mathematic topic in a simple and pleasant way.

MATHFactor assessment criteria

Factors to be assessed during the MATHFactor competition are as following:

- **Topic**
  The topic needs to be related to mathematics, to be original, relevant and meaningful. Where there is controversy or uncertainty for a topic, the presentation should include clarifications. Additionally, the topic should be for the intended audience.

- **Accuracy**
  The presentation should be logical, comprehensible and to use correct language. It should also have a beginning, a middle and an end. Accuracy, clarity and comprehensibility in general are about managing to communicate an idea or topic. Did the message pass? After hearing the presentation, could someone explain the content to someone else?

- **Creativity and charisma**
  Charisma is that unique quality that can be instantly and easily distinguished, but it is always hard to describe: it is about the contact with the audience, the voice-face-body expressivity. Creativity is about the originality of the presentation, the good use of props, the work and effort put in the artistic aspects, etc. This criterion in general is about leaving the audience inspired.

The Judging Panel

The quality of the judging panels is essential not only to ensure the quality of the winner, but also to make certain that the experience is rewarding for all the contestants. Getting the judges' constructive feedback is usually a bonus for the entrants, and it is something they appreciate and value.

Three judges are usually enough for a judging panel but if more people are necessary, then the number should better be an odd one, to avoid a slit committee. Judging is a demanding role that will probably require a lot of time and hard decisions.

The ideal judging panel should have a broad range of experts and different personality types. A prominent and respected mathematician reinforces both the message that the content must be reliable and the rigor of the panel's judgment. Somebody with experience of a live presentation to an audience, like an actor or entertainer, can give invaluable tips on simple techniques like taking a deep breath before going on stage, making eye contact with the audience, and not hiding behind enormous props.

The role of Chair is also crucial. The ideal chair of judges has, in addition to the qualities listed above, the ability to lead a complicated decision-making process, to keep the rest of the panel to stay on time, and the confidence to speak on behalf of the panel to contestants and audience.

Going live or digital?

MATHFactor can be implemented in both a digital environment and a physical one.

When live presentations are the case, participants gather together at a venue to present on stage their topics in the preliminary round(s), and only some of them make it to the finals that are scheduled for another day. Live presentations can be used as an educational tool in local level, for example between the students of a school.

Going digital includes creating a video with a maximum duration of 3 minutes and to upload it to MATHFactor's digital platform. This is an approach that has

the potential to hit a larger audience and to have international participation.

In order to take advantage of all the benefits MATHFactor has to offer, it is recommended to video record the presentations made for a local live competition and to upload them in the MATHFactor platform. In the end, the platform could be used as a meeting point between students, teachers and mathematicians. This way a pool of educational material will be displayed, making the videos publicly available for everyone to watch.

Scheduling MATHFactor for your school

If you are going to proceed with having MATHFactor run live at your school, there are some organizational issues to take into consideration.

In a live competition, feedback from the judges is important and one of the most valuable parts of the experience, especially for candidates who do not make it to the next round. This is why you are advised to allow enough time so that more than one judge can say something to each person. The comments could be recommendations about how the students could have added value in the presentation. Even simple suggestions, like smile more; leave aside too complex props and assessment of the specific qualities of the contestant like “you got me engaged from the start”; “It was very interesting when you did X” can help greatly.

It is always useful to have judging sheets for each judge, with sections to note the content and the participant’s info, as well as to record comments and marks for each one of the judging criteria. It should be noted though that the final decision is more than just summing up the marks– comparing notes is a useful part of the arguing process.

After the presentation, the judges can ask each competitor questions. They may want to probe the content, to test whether a candidate has a good comprehension and if he can respond clearly to follow-on questions. Furthermore, they might inquire about the context of the presentation. Other questions might relate to the participant themselves: why did they participate in the competition? What is their math background? Do they like mathematics in the classroom?

When the judges announce the winners, remember to get photographs of the winner/s to use for further dissemination of the project and publicity.

Although the final will have only one winner, it is also an opportunity for all the finalists to show what they've got. If they perform well it reflects well not only on them but to the competition as a whole. The event must keep the audience amused and interested and should help the participants to bring out their best self.

Also, the idea of having an **audience vote** in a live performance gets the audience more engaged. **The digital equivalent is visiting a website to vote on the videos.** As well as allowing a wider public vote, it is a good way to make the competition popular and promote the finalists. An audience prize should be an attractive gift, but not as big as the one awarded by the judging committee. There is also the alternative option of having the audience vote to contribute towards the overall result.

MATHFactor techniques

As far as performance is concerned, MATHFactor is considered to be interdisciplinary: acting and theatre techniques are combined with skills required for delivering good oral presentations.

Theatrical techniques

Drama conventions are methods that are used to help the instructor in the process of educational drama. They are described as ways to interact imaginatively and to mix time, space and presence to serve the drama purposes while experimenting with different types of theatre. Conventions can be classified in four major groups:

- **Context building**
  Here the effort is focused in setting the scenery and adding info and context such as sound tracking or defining space exercises in order to unfold the drama.

- **Narrative action**
  Narrative action is all about the story. It explores elements like the actions, the time and changes in the plot. Exercise examples include meetings, or a-day-of-your-life narration.

- **Poetic action**
  This means the symbolic part of the drama, through intense use of carefully selected gestures and language such as forum theatre or mimicry.

- **Reflective action**
  This defines the inner thinking that forms the dramatic context with the most profound example being reflective narration or even head voices. The "chorus" in ancient Greek plays had this part in the play.

  The educational drama conventions methodology differs from traditional role-play for many reasons. It is focused in the process itself and not in the final exhibition; this means that participants use it to learn and not to demonstrate some of the skills they have learned. They are actively working on a variety of tasks such as researching, planning and presenting. The teacher or the instructor is not there to give the answers neither to tell participants what to do or what they will learn.

  All students improvise, and there is no script available. This way the same beginning may lead to different outcomes in different groups. The importance of role building is especially emphasized and students are encouraged to discover their own voice and personality.

  However, the most important difference is related to context. When using conventions context is the most important element. What is said and done is shaped by the situations we get involved in and understanding the human behavior in various circumstances.

  Traditional role-play usually works with practicing and rehearsing previously developed skills. In this case, students try to imagine what a different person would say or do in a specific situation, and they use his usual mannerisms such as appearance, and voice to imitate him. Conversely, in drama they experience a specific situation and perform as themselves.

Applied Drama Techniques

On top of presenting a large range of characteristic studies and classroom examples, it is considered to be very useful for the educators to also present a series of techniques associated with the applied drama.

- **Drama games**
  Drama and theatre games are introductory activities and exercises that are used to let the students know what drama is all about. Activities like these tend to not be very intrusive and require high levels of participation.

- **Choral speaking**
  Choral dramatization requests from students to read aloud and assign parts to each participant. It uses texts such as poems or simple rhymes, but also illustrated books. Participants are able to experiment with different voices, sounds, gestures and movements.

- **Tableaux**
  Tableaux has students creating still images with their bodies, focusing on details and relationships. Tableaux are scenes frozen in time and usually involve at least three levels. Participants give emphasis on facial expressions, body language and a focal point. This technique helps in the development of the presentational and the audience skills of the participants.

- **Improvisation**
  Improvisation is dramatizing without a scenario and reacting in response to the environment stimulus. It can be a wonderful introduction to role playing. Students emphasize on position and expression and improve their creativity skills.

- **Role playing**
  Role playing is about playing a character in a situation that might be real or imaginary. This technique can be applied in many areas of the curriculum to support and strengthen comprehension of the content. Below is a list of some common role-play strategies.

- **Reenactment**
  Reenactment refers to acting a scene from a particular historical period or performing a scene from a story. Despite the time period that the scene unfolds, performing is about "now" and things are happening in the present tense. Students interact with a text and develop characters based on that.

- **Extended role play**
  How is a scene continuing after its end? Or what brought things here? A prequel or a sequel of a specific event is used here and cause and effect logic is used and developed.

- **Hotseat**
  Every participant is interviewed while in character. This way further understanding of the role or the content is achieved. Other participants may also contribute by providing extra questions.

- **Expert panel**
  Students do their research and become experts. This way they understand what makes an expert in a particular field.

- **Writing in role**
  An alternative of the above strategies is to ask the students to write something while in character. This method can have many forms like a letter or a monologue.

Oral Presentation Techniques

- **Being nervous: this is something you should work to overcome.**
  It is normal to feel nervous, because people who get on stage are exposed to the public. What you can do, is to observe yourself when you feel nervous and try to work on how to become a better presenter despite your feelings. Do not forget that you can always cover it up. If you have shaking knees, hide them behind a podium, if your throat is sore, have some water close by.

- **Talk about a subject you know**
  Talking about an unknown subject makes you nervous – consequently you get distant and cold with your audience. If you know your subject, you are able to be more friendly and warm.

- **Talk about a subject that inspires you**
  Being inspired by your topic is what makes you so called "natural" speaker.

- **Talk about something you really like**
  Your enthusiasm will be shared with the audience.

- **Prepare**
  The path for a successful oral presentation is to prepare and to rehearse in order to get better.

- **Choose your allies**
  On stage, you can either be alone or you can use tools to assist you. If you choose to use a power point presentation or any other tool, please remember who the presenter is.

  This is you, not the slides behind you. Use the slides to visualize a topic, to create atmosphere, to show some brief notes, but do not let them replace you. If you write all your speech on the slides, then you cancel yourself as a presenter – because people read faster than you speak!

  You can also choose to have some props (some objects that will help you explain something) but keep it balanced all the time.

  Please Note: Despite the fact that the power point presentation is not recommended in MATHFactor and consequently is beyond the point of this text, this section was kept for completeness reasons.

  Also remember to…
  - Have a clear structure of your presentation create a story with a beginning and an ending.
  - Use some theatre techniques like changing the tone of your voice to keep your audience interested.

- Do not squeeze everything there is on the topic in your speech – you will never be able to cover it all.
- Choose what is appropriate to be said for the occasion and keep the rest for another day.
- " Don't let the perfect be the enemy of the good."
- Start on time and keep your time.

- **Writing a speech script – a series of good practices and useful tips**
  Before you start the writing for a speech, it is crucial to understand that writing a speech is very different than writing for print. In speeches, the language must be simpler, direct and straightforward, as the audience does not have a chance to read an argument twice or to come back and reflect a point. It is the responsibility of the speaker to present all points as clearly as possible and the speech as engaging and interesting as it can be.

- **Less is more**
  Keep it short. One of the most influencing speeches in history, is the one of Abraham Lincoln's given in 1863 (Gettysburg Address), and it was not more than ten sentences long. Not everyone can be that short, but for MATHFactor a three-minute speech is all you've got. Do not cut your speech in half; choose carefully what is necessary, interesting and what you think it can add value to your presentation.

- **Know Your Audience**
  Try to understand what your audience is expecting to hear. Make sure there is a strong introduction to your speech (an important tip for written work also) and let your audience know why your speech is important and what it has to offer to them if they stay and listen till the very end. For example: "We will spend the next 30 minutes talking about emergency situations, like how to face a vampire at midnight. This is important, because if you are ready to deal with a vampire, you can face more or less anything that might come across your path!"

  Also, remember that humor is a good way to connect with your audience, so include it in your presentation in a balanced way.

  Moreover, remember that your presentation begins as soon as you take the podium. Your body language, your rhythm, the way you stand, they are all

parts of the show. No matter how important a strong introduction is, do not spend much time on it –the presentation only lasts three minutes, so get quickly to the point.

- **Have the "meat" as a starter**
  In written presentations, each paragraph has a topic sentence with the most important part. The oral equivalent is to have the most important part at the beginning of your sentence, to keep the audience interested. Your goal is to have them wait for the next part instead of trying to get them interested. Have them wonder why you say this or where will that lead? An example to support this coming straight from Hollywood. Imagine in the Star Wars movie, Darth Vader telling Luke Skywalker: "Once upon a time, in another galaxy, in another era, I was different, I had a family, I had a wife and I had a son. This son was you." The actual line of the movie is much more interesting, as its main point is in the beginning: "I am your father."

- **A powerful conclusion**
  To have your listeners satisfied from your speech, finish from the point you started, forming a circle of points and if appropriate give them some homework or some food for thought: "It's unlikely you will meet a vampire tonight, but a natural disaster is an unexpected visitor. So make plans early, have an emergency bag, discuss with your family and be ready to fly away if your friends have kind of pointy teeth".

- **Write an outline**
  Do write an outline, as it is a very useful tool to put things down and organize your thoughts. Then see what you need from your brainstorming and move forward by writing a presentation. Even if you do not want to write down in detail what you want to say on stage, have a structured series of bullet points to give your speech a shape. Keep in mind though that professional speakers such as politicians, entertainers, actors and even stand-up comedians have everything written and scripted, even their little jokes and anecdotes.

- **Understand the way you speak**
  Most people do not write and speak in the same manner, but it is important to keep in mind when you write a speech, you have to write the way you speak and not vice versa. The language is less formal and you should not be afraid to make a sentence less stiff by writing "shouldn't" instead of "should not". Moreover, if you

would say a joke in your everyday conversations or use some slang, you can do it when writing your speech too, but don't go for it if it doesn't suit your personal style. It is important to be authentic and not to sound like you are struggling to make a joke.

- **Practice makes perfect and polishing helps too**
  Do not expect to have everything ready for your presentation from the first draft. Rehearse in front of a critical friend, rewrite, change and improve your weak points, be open to take off some parts and replace them by others. And after you've finalized it, possibly after draft number 25, then remember to rehearse. This is a crucial step, and you must not skip it.

In a nutshell

To sum up, writing an oral presentation is different from writing something that the audience will read. Know your audience and what it expects to hear; write in your natural way of speaking; make it humorous and keep it light (no one can deliver rocket mathematics in full detail in three minutes) and above all rehearse!

Writing down the speech scenario

- **Choosing your goal**
  The starting point of every educational action is to set goals. After all, MATHFactor is also an educational tool and it is meant to help teachers make mathematics more attractive to students. In order to do so, setting goals is crucial.

  The structure of the story should be developed according to two major principles. Firstly, it should revolve around the educational objectives you want to achieve and secondly around the type and the subject of your presentation. This way, you might end up having one structure for a mathematics history narration and a completely different one for a story related to problem solving or skills development

- **Choosing your topic**
  There is a large range of mathematical areas that can be taught and learnt through the use of math-factor activities such as algorithms, algebra and arithmetic, calculus, geometry, topics from the history or the philosophy of mathematics, logic, non-standard problems, numbers and numerical operations, and working with data.

When the time comes to choose your topic, pick a subject that inspires you and serves your educational purpose and find what messages you want to pass. Then try to estimate if what you have thought of, can be realistically delivered in three minutes.

- **Writing tips to remember**

MATHFactor features some unique elements, like the three minute restriction and it has some tips you can follow when writing. One of them is that the whole speech length should not be bigger than what can be included in one piece of A4 paper, using Times New Roman fonts of size 12.

Moreover, for a successful presentation, the "herd of cows" anecdotal pattern should be followed. When a shepherd wants to lead his herd through a particular path, he makes sure that all the cows will follow the same path. Similarly, when writing and preparing a presentation you need to make sure that everything, even the tiniest element should follow the same path; meaning that everything should be aligned with the presentation's goal and the main message.

Finally, an important thing you should keep in mind is that there is a big difference between written and oral communication. After you have finished writing down your story, you should read it again and you should try to get rid of every literary element and every word or phrase that is not used in everyday oral speech such us adjectives, adverbs and sophisticated vocabulary that do not serve a purpose other than to enhance a text. Additionally, you should avoid using mathematical terminology. You should remember that in order to use a term you need to explain it first. If you have no time to explain it, then it should not be included.

Writing the script

The beginning

Everything begins with the main outline of the script. But how do you start writing it down? The script is a complex process, and it is going to be built through a series of different stages, but the starting point is always the most important. The teacher can help the students during the first stages by using

creative writing exercises. In the following section, we are going to present two of these methods: the writing burst and the different point of view.

- **The writing burst**

  A writing burst is a 10 minute writing exercise. The teacher gives the students a topic and asks the group to start writing for 10 minutes without worrying about the quality and the appearance of their work.

  The thought of writing a short story on your own can be very intimidating. It is much easier to set a timer for 10 minutes and to begin writing without stopping or looking back.

  How does this way of writing helps? This method is usually used by journalists or by writers, when they have very little time to write an article or when they want to create the mood for writing a bigger piece. Sometimes this material can be used as a starting point for a work. Moreover, writing bursts help people to generate innovative ideas, because they write without stopping or looking back and they do not have to worry about spelling or grammar mistakes.

- **The different point of view**

  It is funny to think about narrating the “Three Little Pigs” story as the big bad wolf. Write a title like “the true story” and start working on presenting the story from this point of view. Or how about writing the true story of “Cinderella” from the point of view of the two bad step-sisters?

  And now let us imagine how these can apply in mathematics. For example, imagine the birth of number zero from the point of view of other numbers. All the other numbers think it has no value at all, until it pairs with one of them. Likewise, spare some time thinking about the Pythagoreans. Besides the famous theorem, research the true story of the Pythagoreans, this strict community. Will a rejected student stay alive to tell his story? Or imagine, as happens in Flatland, a rectangular telling the most unlikely story of his 3-dimentional adventure, in prison, alone and disconsolate because nobody believes it.

Let the children consider what they know about the mathematics topic of their interest and let them imagine and write another version from another point of view.

An inquiry about the topic should follow and the results could be announced in the classroom. This procedure could bring new innovate ideas on the table and inspire students.

Script building

After gathering all the necessary information, building the story comes next. The five "W"s are the key to unlock any difficulties you might encounter in putting your ideas in order: where, when, what, who and why?

- **Where and when did the play happen?**
  The answers here could vary from historically accurate (in the library of Alexandria at 200 BC) to fully imaginative (in a planet hundreds of light years away).

- **What happened (exactly)?**
  The facts should be put here in order to unravel the story.

- **Who did all these?**
  Is the main character going to be a historical person? Is it going to be an imaginary one? Or it is not going to be a person at all, but a personalized math symbol or idea? For example a function that is depressed because it has its concave down and decreasing.

- **Why did this happen?**
  The aftermath and the moral of the play can be found by asking why. What were the character's motives to act like this? Did the general situation accelerate things and set the plot into motion? What about the politics or the social factors of the time?

- **How did this happen?**
  This is a bonus question, giving us space to further develop and flesh out the story. It is the question that asks about details and ideas and the question that takes the writer deep into the heart of the action.

After having created the story, the next step is to trim it down. Remember you only have three minutes in your disposal. Keeping it short is one of the little success tips that are not always easy to follow, as no one is happy to see the text he/she wrote about to be cut. However, this step is necessary and the best way to correctly identify what should stay and what should go is to read loudly the text to an audience (in this case the rest of the class). There will be spots where even the narrator will want to pass on more quickly.

Rehearsals and Preparations

With the script ready, it is time to go on with the rehearsals and to set the stage for the performance. Decide on the props you will use, if the are any, and rehearse following the techniques already mentioned above.

Adapting a scenario

In some cases, when time is limited, or if there is a play that is encouraging the students or the educator, they could consider adapting a scenario. This could also be the case in adapting a book or movie to a play.

The first thing you must think before any adaptation is the intellectual property rights. Usually every author of the original text material holds the copyright. This means they have the right to say if a play can or cannot be made based on their material. If the answer is yes, how much will it cost?

The legal and correct thing to do is to check and get contact with the writer, so you can start the procedure of buying or optioning the rights. Sometimes if the material is used and adapted for educational purpose, dispensation is free.

Additionally, because of the copyright expiration, if you are interested to adapt a text written in the 18th century, the work is considered to be “in the public domain” and you have not the obligation to secure any rights at all.

However, a MATHFactor presentation has its own rules; you need to squeeze all the information you derived from this material in just three minutes. This alters the character of the material and usually solves every problem one might have with copyrights. Probably the only exception to this is when you use a popular

song as a background, because the MATHFactor presentation will be uploaded online, and that might provoke a copyright conflict.

After clearing out the copyrights, the question is how to adapt the story. The methodology of work is the same with the one needed to write a story. The same techniques apply here: keeping it short, choosing what to say and align everything with the presentations' main messages. Use the where, when, what, why, who and how keys to unlock the story. Give emphasis, promote your unique style, let some humor in, raise or lower your voice and have fun!

Section A5. Competitions and Events

Mathematics and competitions can be combined in many ways; the MATHFactor competition is one of them. In this chapter, we are going to lay down the guidelines on how to organize such a competition or event.

Planning and Administration

A well planned event will save you time, resources and money. You should be able to break down the key roles and tasks of each of your team members so that you can proceed efficiently. Determine your target audience and if the competition/event is local, national or international. After identifying your audience, try to collect contact details (emails, addresses etc.) in order to create a database that will help you to send invitations, information, promotions, and so on. It should be noted that decision makers (ministers of education, school directors, national agencies etc.) can play a critical role for the dissemination of your competition/event. If the number of the participants is great (more than 200 students), it is better to separate the competition/event in more than one phases.

Venue and date

Finding a venue and setting a date are probably the first major difficulties encountered when organizing a successful event. It is hard to continue with any other aspect of the overall planning until you overcome these two key hurdles. It is recommended that you examine these two questions at the same time: select an ideal set of dates and search potential venues in order to find the best possible fit.

It is crucial to pick a suitable date for your competition/event to avoid competing with other events in your area that will attract the same audience. For achieving the best possible scheduling, you should check that your event/competition date does not conflict with any other famous events. Additionally, you should take into consideration holidays, university and school calendars to avoid scheduling the competition/event during exams.

Choosing a venue is one of the most important steps in organizing a competition/event. A poor choice can undermine even the most well-planned events while

a good one can make a good event even better. When looking at a possible venue, you should take into consideration the potential cost. Be sure to check all venue costs (venue, security, catering etc.) to verify that it will fit your budget. Additionally, make sure that it meets all your needs. For example, you would probably need a venue that has enough parking lots, a presentation room with projector and having a suitable size for your event. You should also take into account that if your event lasts more than one day attendance may vary, especially in weekends, so you will have to manage your space accordingly.

Budget

It is the organizing team's responsibility to keep track of all event expenses. To begin planning your budget, firstly you should consider how many attendees you are expecting as this will have a direct impact on your choice of venue, supplies, food and equipment. As soon as you have a clear idea of the scale of your competition/event you are set to move forward. Even though each event might be different, you should be able to identify and break down your main costs. Indicatively, you should be able to estimate your expenses by taking into account the cost of the:

- Venue
- Food and Beverages
- Supplies and Equipment
- Marketing/Promotion
- Travel and Accommodation
- Gifts and Memorabilia

Additionally, where applicable you should aim to use volunteers in order to avoid hiring professionals for the tasks that do not require a high level of expertise. Also, a good way to tackle some of the costs is to find sponsors that will be willing to share some of the expenses.

Promotion

Promotion is arguably the most difficult and time-consuming aspect of organizing an event. It is also imperative, as it is in your best interest to promote your event in order to maximize attendance. This can be done in many ways with varying costs. You will be forced to be proactive, outgoing, and you should be ready to make some new connections. When promoting your event you should have

a clear idea of your target demographic area and try to focus your efforts via channels that are more accessible to them. The more variety and imagination you will use in your dissemination efforts, the more rewarding will be the results.

MATHFactor advertizing 2013

The use of social media is highly recommended as it is free, and it allows reaching an audience, which it might be otherwise inaccessible. Additionally, depending on the budget you should consider promotion through radio and television. You may keep contact with the media through a press-conference.

LE-MATH Press conference

You should also print posters and leaflets and distribute them in schools, universities, NGO's, etc. that might be interested in your event. In many cases, it is highly recommended to create a webpage or advertise through your organization's webpage, by providing specific information for the event (maps, cost of attendance, FAQ etc.). You should remember that having a well-organized and fun website with rich content is the easiest way to convert traffic into registrations.

Speaker and Judges

A high-profile speaker is always a very good way to create buzz about your event. In some cases, it can also help you promote your event/competition and even sell tickets. Depending on your event you should cap the total allotted speaking time appropriately and manage your time effectively.

Having a panel of judges to evaluate the final presentations is a great way to provide added value to your event. Similarly, with key speakers you should aim to have at least one or two high-profile judges, in order to add to the credibility of your competition/event and to serve as an extra mean of promotion.

Who is the best? The Jury is working, MATHFactor Europe 2014 Competition

Section A6. Being filmed or recorded - developing communication skills

Being in front of a camera

If you have never presented in front of a camera before, then the first time can feel a little nerve-wracking and unnatural. Do not worry too much though, as it only takes a little bit of practice and preparation to start feeling more comfortable and confident. This chapter points out some tips for presenting in front of the camera to help you to feel more confident and prepared.

Relax. If you feel uptight, you are going to look and sound uptight! If you can, try to think and discuss possible questions and answers before start filming. This will help you feel better prepared and more comfortable.

Prepare and practice your script. It is important to know your subject from inside out so that you feel confident talking about it, even if you are very nervous. There is a difference between knowing your subject well and reciting your script word for word. If you recite everything word for word then you run the risk of sounding robotic. You should make sure that you can speak about your topic confidently and naturally.

Talk Slowly. It is normal to feel nervous, especially if this is your first time in front of the camera. Your adrenaline is flowing, and your heart is beating a little faster, and you start talking much faster than you normally do. If you think you are talking too fast, you probably are. Speak clearly and practice your tone of voice and make sure that you are not mumbling. Remember that it is imperative to vary your tone and not your volume. Use the tone of your voice to stress words or sentences and make sure that you split your presentation in sections, pausing after every section or end of a sentence.

Use simple language. If possible, avoid complicated technical terms and acronyms that need explaining. Avoid words, terms and phrases that non-expert audiences might not use in everyday dialogue.

Know where you need to look. Although you may need to present directly to the camera, the audience will be looking at you through the camera lens. Talk to your audience as if they are right in front of you.

Have control of your expressions. Remember that when you are presenting in front of the camera, people will be able to see all of your expressions very clearly. If you are used to presenting live to large audiences rather than in front of a camera then, you may not be used to this. Make sure that you are focused and in the right frame of mind before you start presenting.

Unless you are delivering bad news, you should smile. Smiling not only warms up your visual presentation, it also warms up your voice.If you want to appear approachable, you can nod your head while speaking. If you want to appear credible, keep your head still and slightly drop your chin at the end of your sentences.

Avoid uncontrolled hand gestures and other body motions. A few slow and deliberate hand gestures are ok, but avoid quick, broad and sweeping hand gestures. It is difficult for the camera operator to "keep up" with quick motions.

Remember good posture! Your voice can be saying one thing, and your body language can be saying something entirely different. If you want to show an object, raise it slowly and tilt it slightly towards the camera to avoid glare from lights. You may wish to practice before the final shoot.

Movement on camera can be very distracting. Watch newscasters or actors and you will see that for the most part they are very still. This does not mean you should not use gestures. Just be careful to avoid movement that does not have a purpose.

All movement is exaggerated on camera. If you want to lean forward to show interest, just make a slight movement. Avoid frequently moving towards and away from the camera.

Be Confident. Even if you are a little nervous or unsure about how you will perform in front of the camera, act confident – it helps your presentation.

Don't rush it. If you are feeling a little nervous then you might feel tempted to rush through your script without pausing. Make sure that you speak clearly and naturally and pause to collect your thoughts throughout your presentation.

Avoid bad grammar, slang and swear words. They distract from the presentation and can lessen your credibility with the viewing audience. After all, the information you are sharing is important.

Be conversational and be yourself! Put a little variety in your voice. It might help to think about how you talk to another person on the telephone. Show an interest in what the presentation is all about. Talk as though you are speaking with a friend.

Do not rely on just the light from the room you are filming in. Instead, use daylight balanced light to even out your skin tones. Lighting you from the front will keep shadows from falling on your face.

Whatever is visible behind you can be visually distracting. Be aware of what is in the "shot" and be sure to keep your background as pure and simple as you can. A mess behind you or on a bookshelf can send a negative message about you. If you have a blank white wall, consider adding a plant for some visual interest.

Make sure your clothing is appropriate, well-pressed, clean, and well-fitting. A small stain or wrinkle, which may seem like no big deal in person, can be distracting on screen. Try to wear solid colors, but watch out for black and white clothing, which can be problematic. Clothing with small prints or patterns can "vibrate" on video. Also, avoid jewels that can clank against the microphone and make noise.

Whether you are a man or a woman, beware of face shine. A shiny face or forehead can be distracting. You can use blotting papers or a light powder to limit the effect.

Lighting can affect how your makeup looks on camera. If you have bright light shining on you, it can wash out the appearance of your makeup. Test how your makeup looks by videotaping and reviewing.

Make sure your hair is not sticking up in a distracting manner. While you should avoid shine on your face, shiny hair can look quite nice. There are lots of products that will help you achieve this effect.

Be careful to avoid slumping in your chair or standing with poor posture. When seated, you should sit on the forward third of the chair bottom with your legs at a 90-degree angle and your feet flat on the floor. Sitting this way keeps

your diaphragm free so that you can breathe properly and speak dynamically. It also gives you a firm base, thereby reducing any unnecessary movement.

When standing, place your feet hip-width apart, knees slightly bent, and arms comfortably at your side. To stand up straight, imagine that there is a string attached to the top of your head pulling it up.

Cut – take two. Remember, you don't have to get it right the first time round. You can always videotape your presentation over and over until you get it right. If you know how to use video editing software, you can always edit different scenes to get your video looking perfect.

Of course, always be yourself and have fun!

Section A7. Attracting media attention - for motivating pupils and promoting mathematics

Attracting media coverage. Every person wishing to promote something (product, service, presentation etc.), believes that their presentation is the best thing, idea or concept since the invention of the wheel.

However, journalists, bloggers and press in general are bombarded with hundreds of press releases every day, promoting the ground-breaking, revolutionary, mind blowing features of their presentation, service or product.

So why would someone pay attention to your presentation and not someone else's.

- **Tell them what they want to hear, not what you want to tell them**.
  You should try to look things from a different perspective, especially if you have a diverse presentation (covering different subjects). Understand that what you might think is important may not be what the media usually likes to cover. So think like a journalist, not like a Mathematician.

- **Understand that editors and writers have very tight deadlines.**
  Today's news cycle is extremely fast paced, making the pressure to continually create new content nearly unbearable. The more complete your press release is, the less research a reporter must conduct. Put your story within reach, and it is more like to get picked off the tree.

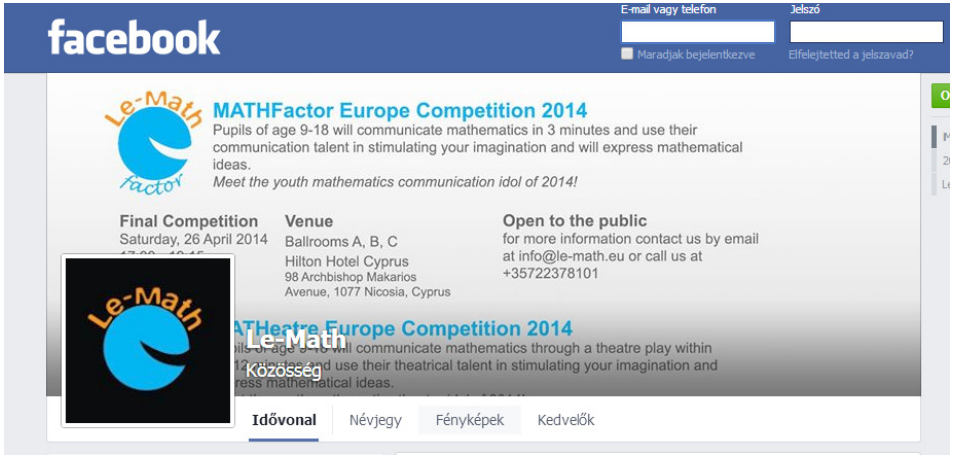
- **The copy – paste syndrome. Yes, journalists do copy – paste**.
  If you supply a well written, interesting story that is ready to be published, you more than double your chances. Make sure you provide a ready to publish document, with correct grammar and spelling, written as if the journalist is interviewing you.

- **Photos and other media.**
  Make sure to include interesting photos or other media if you send your press release to blogs, news portals or TV channels.

- **The more the merrier.**
  Do not just count only on one or only to a few media. The more press releases you send out, the more are your chances for your story to be published.

- **Spread the word evenly.**
  Do not focus on only one type of media such as newspapers or radio. Send your press release to as many types of media as possible. Make sure to include digital media (news portals, blogs etc.). Remember that it costs money to have your press release printed in newspapers and magazines. The same applies for TV and Radio when they advertise you. On the other hand, digital media might cost you significantly less or even nothing.

- **Publicize yourself.**
  The power of social media is open to just about everyone. You can publish your own press release on more than a handful of social media and blogs by yourself. Get people to share your article and ask your friends to promote it.



Use social media platforms

- **Follow up.**
  Do not just depend on a plain email. Call and talk to the journalist. Make sure they got and read your press release. Ask if he liked it and if he intends to publish it. If he does, find out when and make sure to thank him. If he does not, find out the reason, you may be able to get pointers on what to do to get your press release published in another outlet.

PRESS RELEASE

Le-MATH
Learning mathematics through new communication factors
A new European Commission funded project (Comenius MP)
running from November 2012 to October 2014
526315-LLP-2012-CY-COMENIUS-CMP

Many pupils as well as parents unfortunately consider mathematics as a difficult and boring subject. Instead of studying mathematics (and other subjects) many pupils prefer to spend most of their time watching TV programmes or playing electronic games or exchanging messages with their mobile phone, exchanging pictures, exchanging videos, competing etc. One way to bring pupils back to the "playing field" of education is to use similar tools (weapons) like the "opponents", that is to communicate the learning of mathematics in a non-traditional way, like a game through theatre or competitions similar to the well-known X-Factor and other.

Le-math press release

- **It may be necessary to educate the media.**
  Especially if your press release contains technical information, difficult mathematics or methodology that journalists may not easy to understand.

- **Make sure you know all facts and figures.**
  You must know your subject inside and out when approaching journalists, because journalists typically work under very tight deadlines and they often need information quickly. In many cases, media coverage could be lost because essential information was not available on time.

- **News cycle patterns**
  Familiarize yourself with media cycles (newspaper print schedule, TV/Radio program schedule etc.), so that you can plan your events and press releases accordingly.

- **Press contact information**
  Be sure to have e-mail and fax numbers for your target reporters. A good press release doesn't do any good if you can't get it to the right person.

- **The general desk won't do – target particular journalists.**
  Don't send your media releases straight through the general news desk. By doing this you'll be running the risk of having your story overlooked. Target specific reporters to increase the chances of having your story covered,

especially social and community affairs reporters in bigger news outlets. Reporter details can be found on an outlet's website, or through a quick phone call to the outlet.

- **Be available**
  Make it easy for a journalist to contact you to follow up on a story – this is as simple as providing mobile phone number. Being easy to reach can be especially useful if a journalist is trying to contact you.

- **Get the inside word.**
  Media training does not have to be formal or expensive – it can be as easy as asking a journalist for advice on how you can become better at attracting coverage. Approach a journalist with a request for a drink and a chat about attracting coverage, and be sure to mention you are a student/teacher. Be respectful of their time.

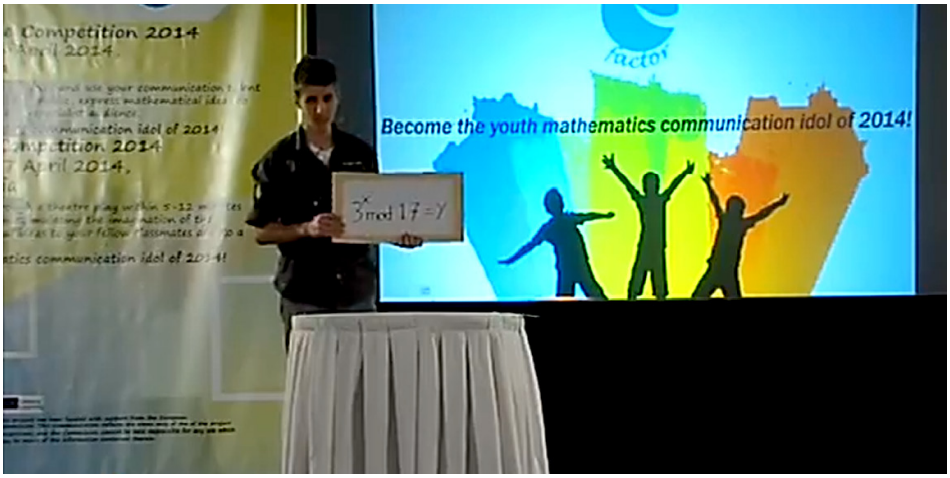
- **Do not use the phrase "No comment".**
  You might be under the impression that by saying, "No comment" to a journalist it will save you from answering any hard questions. This is wrong. When no comment is offered to journalists, it is like a red flag shown to a bull.

- **Link to a major event or hot topic!**
  Tie your idea to current news, event or hot item of public interest.

PART B MATHFactor and Mathematical Competence

Section B1. Approaches of using the MATHFactor methodology in mathematics

In the **GENERAL REMARKS** and **PART A** of these Guidelines, we presented thoroughly the advantages of using a theatrical approach in mathematics learning. We argued that MATHFactor is a motivation tool that promotes communication skills and improves mathematical learning. Additionally, we explained the various types of activities and the approaches for exploiting and linking MATHFactor to the curriculum. Finally, we analyzed the role of the teacher and the student as a presenter and the importance of the theoretical background in the proposed activities.



The role of the student as a presenter

For this reason, a number of supporting tools have been produced and accompany the present manual in sections B1 and B2. Subsequently, we have to ensure that these elements are in line with the responsibilities of a teacher.

These supporting tools provide numerous examples of practices in this area. Additionally, there are analyses and comments on many of these scripts or stories associating them to the areas of mathematics they are referring to, such us the

age group of the pupils that are appropriate for and the pedagogical outcomes/ goals that can be achieved through them.

From the presentations in **PART A** it becomes obvious that the MATHFactor approach can be implemented as following:

- In presentations that are implicitly supporting the mathematical curriculum. Such activities are formally prepared and usually are taking place:
    - Trough presentations that are to be parts of the activities in an event in a school.
    - Through participation in a competition.
    - Through a specially designed presentation in a class.

• In presentations that are explicitly and immediately supporting the mathematical curriculum.
Such activities are usually part of the everyday activities in mathematics classrooms and are prepared in simple terms and with restricted use of demanding outfits, effects, etc. They can be prepared and presented:

- Through the adaptation or the preparation of a specially designed script by the teacher, in order to enhance the learning of a concept, process or other mathematical activity that is part of the syllabus for this particular age group and during the appropriate time, by taking into consideration the background of the pupils and the associated mathematical objectives.

- Through the adaptation or the preparation of a specially designed script by the pupils in order to enhance the learning of a concept, process or other mathematical activity that is part of the syllabus for this specific age group and during the appropriate time taking into consideration the background of the pupils and the associated mathematical objectives. Obviously this preparation should be under the assistance of the teacher (perhaps as part of the project).

  Some examples of the latter (explicit use) are presented in B4.

Section B2. Using the Manual of Scripts for MATHFactor

The Manual of MATFactor scripts contains 37 original scripts developed by the Le-Math project partners in order to give different ideas to teachers and pupils about the new method introduced here. The variety of ideas will help users of different age groups and mathematical backgrounds to find a proper example, which can be used directly without any change of the script (the scripts are ready to use) or to make adaptations when the teacher or the student feels it is necessary. Most of the examples are offering subjects that are very popular in Mathematics, and at the same time they help students to understand the application of Mathematics in real life situations or to understand better the mathematical reasoning. We recommend that teachers and pupils read and discuss the content of some of the scripts before deciding which one to apply and in what form they will apply it. This reading is aimed in the start of the preparatory work, and the pupils and teachers can try to collect other information related to the subject(s) they have chosen, to find out connections to their own experience, country or area, history or people.

Once a script is chosen, the students should study it very well in order to understand it thoroughly and in order to be able to present it in public. They should be prepared to become “masters” of a given problem or question, as this is the best way to get the necessary confidence to present the subject. They will become the “teacher” for a few minutes, and they have to explain it to their classmates in a manner that will not only help them to understand, but that is also enjoyable. The information that a student gets from a classmate is easier to digest as they speak the same language.

The pupils should try to use their own words. If an expression seems unusual, or too complicated, they should try to find another more simple, or maybe add some explicatory remark (e.g. if in a script it is mentioned the orthocenter of a triangle, and they are afraid their classmates will not understand it, or will think the presenter himself is unfamiliar with it, they should add an explanation, like: “You know this is the point where the three altitudes of the triangle meet.”).

If the script contains a proof, the pupil presenting it, should understand all the details of the proof and he or she must present in such a manner, that the listeners can follow all the steps of the presentation. The presenters should keep

eye contact with the public. Of course the presentation in a classroom differs from one in a public show, or from a competition where the feedback of the spectators cannot be taken into account.

The manual of scripts offers a good support for both the teacher and the pupils, but the analysis of the scripts is mostly aimed towards the teacher. First, they are able to decide from the script analysis, if a given script can be suitable for a given age group, for a given subject, and if the method presented through the script fits in the curricula at the given moment. They should choose the right script and to recommend it to their pupils. Often the first reading of the script gives a good start and offers ideas. The teacher and the pupils might end up making an entirely different script, if they consider it more suitable for a given learning situation. In this context, the teachers should use the manual of scripts as a generator of ideas rather than a collection of compulsory examples.

For example, we attach here a MATHFactor script which served as a starting point for a presentation in the MATHFactor Europe 2014 competition, and won the first price for the age group of 9-13. An analysis of the script is included here as well, to give an idea of what kind of support the teachers are offered.

The script model.

Eurobanknotes

- **Preparation**
  Teacher trainees are introduced to the concept of the competition MATHFactor Teaching and learning mathematics through mathematics communication activities. They discuss the ways in which mathematics can be made more interesting and entertaining to pupils and students and discuss the idea of the proposed methodology.

- **Realization**
  They are shown video recording with Ema's performance in MATHFactor.

The Scenario
The student comes on stage. She has a model of 2 Euro banknotes to use in her presentation. She introduces herself to the public and starts her presentation.

Text: *Hello, my name is Ema, I am 13 years old and I am here to tell you something about Euro banknotes. As you know, Euro is the currency used in many European countries. The banknotes are produced from pure cotton fiber which increases their durability and gives them their characteristic touch and smell.*

There are many ways in which these banknotes are protected. There are:

- Holograms
- Watermarks
- Digital watermark
- Infrared and ultraviolet watermarks
- Magnetic ink
- Microprinting

However, there is one extra protection connected to mathematics called the CHECKSUM. It is connected with their unique serial number. [takes the first specimen of banknote and holds it, in a way, visible to the audience] The first letter in this serial number stands for the country the banknote comes from. For example Z stands for Belgium, Y for Greece, X for Germany and G for Cyprus. Now back to the checksum. Each serial number on a banknote is created in such a way that if we substitute the first letter in a number by its position in the alphabet (e.g. A is 1, B is 2, C is 3 etc.) the sum of all digits divided by nine has the remainder of 8.

Let me show you. [Points at the banknote she is holding whose serial number is M50027558701]. The first letter in the serial number on the banknote is M (the banknote is from Portugal). M is the thirteenth letter in the alphabet. So 13 + 5 is 18 + 2 is 20 + 7 is 27 + 5 is 32 + 5 is 37 + 8 is 45 + 7 is 52 + 1 is 53 = 5x9 + 8

And another example. [Takes another banknote with the serial number V91782110236] The first letter in the serial number on this banknote is V (the banknote is from Spain). V is the twenty-second letter in the alphabet.
So 22+9 is 31 + 1 is 32 + 7 is 39 + 8 is 47 + 2 is 49 + 1 is 50 + 1 is 51 + 2 is 53 + 3 is 56 + 6 is 62 = 6x9 + 8.

See? It works. Now you can always check that the banknote you get in a shop or bank in not fake.

- **Post-task**
  Teacher trainees discuss the video with respect to:
    - Mathematical content
    - Presentation
    - Language

  They work in pairs to propose possible improvements to the presentation.
  They develop a lesson plan ii which they would make use of Ema's presentation.

- **Follow-up**
  Task for teacher trainees – think of other codes that are used in everyday life and have mathematical background. Think how your student could be presenting it in an entertaining way to the public. What language, what materials would be needed? Who would be the target audience?

Appendix: Banknotes use in the presentation

ANALYSIS

Math Topic: Euro-banknotes

Age group: 9-13

Knowledge background:
Understanding of basic numerical operations and division with remainder. It does not require any other specific mathematical knowledge.

Knowledge acquired:
The ability to follow mathematical instructions presented in a verbal form.

The activity develops interdisciplinary and intercultural knowledge. Through the banknote code, pupils learn about EURO, the common currency of EU, as well as about individual EU countries. At the same time, knowledge of physics and chemistry is developed if considering some other protective elements.

The topic can be quickly developed further e.g. by showing the use of control digits in other real life examples as bar codes for goods, checking sums for personal documents, ISBN for books or ISSN for journals.

Skills acquired:
The story shows possibilities of the use of mathematics in real life. It might be surprising for many people that mathematics is applied even in such simple objects like banknotes. The problem can motivate students to look for other similar examples of "nonvisible" use of mathematics in real life.

The preparation and presentation require developing pupils' problem solving skills. The problem can be presented to students in the form of a puzzle (asking them to calculate the last digit of a concrete banknote) or as a game with the goal to find false banknotes in a set of banknotes.

The activity also develops the ability to mathematize the situation described in words and to work precisely. The calculation of the control digit supports and develops calculating by heart. The problem provides pupils with an intermediate feedback because it is sufficient to uncover the last digit and check if the calculation was correct.

Preparing the presentation of the problem (scenario, acting and use of visual tools etc.) develops pupils' Communication skills. The activity makes mathematics more popular by showing that even in real life results of "simple" mathematics can play an important role.

Section B3. Exploiting Other existing examples

Besides the manual of scripts the users of the MATHFactor method have various examples of ideas in the collection Good examples, made up by the project partnership, which contains an analysis fitting in the general methodology of the guidelines. Of course these examples are meant to reflect the way the experience of a larger community of mathematics teachers is related to the new methods introduced by the project.

However, the project partners do run a series of activities, (MATHFactor competitions, Euromath Conferences), which have already produced a large database of examples, all accessible through the project's web page. The user of the guidelines is recommended to go to the web page of the project to become familiar with the structure and its contents. The users of the web page will be able to watch online, hundreds of videos in different languages. These videos contain MATHFactor (and of course MATheatre) presentations of high quality. Most of them have been appreciated by national or international juries, and were selected for the MATHFactor Cyprus or MATHFactor Europe Competitions.

The users of the web page will see unexpected, but interesting ideas; they will see how creative our pupils can be. Some examples of students' presentations are the following: Using a magnetic board to explain the idea of snow-flake curves, to bring with special, "mathematical" cakes, made by the participant to illustrate the idea of packing, to show a big copy of a Euro-banknote to explain coding, to wear magic hats, historical costumes etc., to support the story presented there. The reader is expected to identify the ideas mentioned above by watching the videos of Euromath 2014. Additionally, he can also find resources in the project web page at www.le-math.eu.

Section B4. Developing ideas of pupils or teachers in the spirit of the MATHFactor approach

EXAMPLE

Introducing Mathematical Induction through the MATHFactor Approach

Part of the Curriculum: Introduction to Mathematical Induction
Age Group: 16-18 years old
Goal: To explain the Process of Proof using mathematical Induction by

- Identifying a conjecture that has to be proved
- Identifying the essential prerequisites to be secured in order to apply the process

Preparatory work

The teacher asks two pupils to make presentations, in the spirit of the MATHFactor approach, based on the following stories:

The students are expected to demonstrate communication skills so that their fellow students would be happy and comprehend the process. For this reason, various expressive approaches should be employed to be as vivid as possible. The stories offer a lot of opportunities for this.

Story 1

John and Mary want to go to heavens. For this, they have to use a ladder through which they can climb each time one rung. George observed that the critical stages to achieve this are:

Stage 1: Each climber can move to the first rung

Stage 2: Given that a climber has reached the K-th rung he/she can move to the next (K+1)-th rung.

What is the conclusion of this?

What principle can we deduce?

Story 2

The Second Labour of Heracles

The Lernaean Hydra (Greek: Λερναία Ὕδρα) was an ancient serpent-like water monster with reptilian traits. It possessed many heads — the poets mention more heads than the vase-painters could paint — and for each head cut off it grew two more. It had poisonous breath and blood so virulent that even its tracks were deadly. [1] The Hydra of Lerna was killed by Heracles as the second of his Twelve Labours. Its lair was the lake of Lerna in the Argolid.

Given that the Lernaean Hydra had seven heads when Herakles visited the lake, in order to kill her. Given that each time, he was cutting one head then two new heads were born out of the cut. Assuming that Herakles was able to cut all the heads each time he used his sword present a conjecture of the number of heads after he used his sword n-times where n is a positive integer. The presenter is expected to:

- Identify the formula for the conjecture
- Present an argument identifying the necessary steps in order to prove the conjecture
- Explain why both steps are necessary for the final result

REFERENCES

Bonwell, C.C, & Eison, J.A. (1991). Active learning: creating excitement in the classroom. *ASHE-ERIC Higher Education Report*, Washington, DC: George Washington University, School of Education and Human Development.

Cobb, P., Wood, T., & Yackel, E. (1994). Discourse, mathematical thinking and classroom practice. *In contexts for learning: Sociocultural dynamics in children's development*. New York: Oxford University Press.

Dochy, F., Segers, M., & Sluijsmans, D. (1999). The use of self-, peer and co-assessment in higher education: A review. *Studies in Higher Education*, 24(3), 331-350.

Lampert, M., & Cobb, P. (2003). Communications and Language. In J. Kilpatrick, W. G. Martin, & D. Shifter (Eds.), *A research companion to principles and standards for school mathematics* (p.p 237-249). Reston, VA: National Council of Teachers of Mathematics.

Ministry of Education and Training. (1997). *The Ontario curriculum: Grades 1-8 Mathematics*. Ontario: Queen's Printer for Ontario.

Ministry of Education and Training. (2006). *A guide to effective instruction in mathematics, Kindergarten to grade 6, Volume 2: Problem solving and communication*. Ontario: Queen's Printer for Ontario.

National Commission on Teaching and America's Future. (1996). *What matters most: Teaching for America's future*. New York: National Commission on Teaching and America's Future.

National Council of Teachers of Mathematics, Algebra working group. (1998). A framework for constructing a vision of algebra: A discussion document. In National Council of Teachers of Mathematics & Mathematical Sciences Education Board (Eds.), *The nature and role of algebra in the K-14 curriculum: Proceedings of a national symposium (pp. 145-190)*. Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics.* Reston, Va: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, Va: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, Va: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional Standards for teaching mathematics*. Reston, Va: National Council of Teachers of Mathematics

National Research Council. (1998). *High School mathematics at work: essays and examples for the education of all students*. Washington, D.C: National Academy Press.

National Research Council. (1996). *National science education standards*. Washington, D.C: National Academy Press.

National Research Council, Mathematical Sciences Education Board. (1989). *Everybody Counts: A Report to the National on the future of mathematics education*. Washington, D.C: National Academy Press.

Neelands, J., & Goode, T. (1998). *Structuring drama work: A handbook of available forms in theatre and drama*. Cambridge, UK: Cambridge University Press.

Silver, E.A., Kilpatrick, J., & Schlesinger, B.G. (1990). *Thinking through mathematics: Fostering and inquiry and communication in mathematics classrooms*. New York: College Entrance Examination Board.

Silver, E.A., Schwan S., & Nelson, B.S. (1995). The QUASAR Project: Equity concerns meet mathematics education reform in the middle school. In W.G. Secada, E. Fennema, & L.B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 9-56). New York: Cambridge University Press.

Smith, M.S., Hughes, E.K., Engle, R.A., & Stein, M.K. (2009). *Orchestrating discussions. Mathematics Teaching in the Middle School*, 14 (9), 549-556.

Verhoeff, T. (1997). *The role of competitions in Education*. Eindhoven, Netherlands: Faculty of Mathematics and Computer Science.

ACCOMPANYING TOOLS/MATERIAL

In the process of adopting the MATHFactor approach the user can find a broad range of examples that can be of great help either for approaching a particular area of the mathematical curriculum or for enriching his/her lesson or finding ideas for participation in competitions or preparing a communication performance for a particular occasion relating to mathematics. The present project has prepared some packages of such examples and are provided as part of its outcomes. The user can exploit these tools/material in order to enrich his/her store of resources.

These tools/material are organized as following:

MF-Tool 1: Le-MATH Manual of Good Practices (link to www.le-math.eu)

MF-Tool 2: Sample videos of MATHFactor (DVD and link to www.le-math.eu)

MF-Tool 3: Manual of Scripts for MATHFactor (publication and link to www.le-math.eu).

ANNEXES

ANNEX A1

Table of Contents

0. Description

In this annex one can find a structured analysis of the scripts in the publication "Manual of Scripts for MATHFactor" (ISBN 978-9963-713-12-7). The idea is to use the Manual without the analysis in order to be approached from a pedagogical point of view and used for practice without reference to the Guidelines book above. The analysis is mainly for the use by teachers teaching mathematics to pupils of age 9-18. Even though the analysis indicates a suggested age group, the user may find it useful for different ages, depending on the local curriculum used.

1. A beautiful trip to the beauty of Φ

Math Topic: Golden ratio

Age Group: 9-13

Knowledge Background: Number division, Analogies

Knowledge Acquired: Properties of the Golden ratio

Skills Acquired:

The preparation and presentation required for this MATHFactor develops the understanding of the golden ratio.

Mathematical Modeling Skills – acquired in order to apply the properties of the golden ratio in the human anatomy and in famous buildings like the Parthenon.

Visualization Skills – developed as the student shows the parts of the body that need to be measured in order to find the golden ratio.

The human body, the rose, the coral and other God creations are beautiful because their analogies are equal to the golden ratio. The Ancient Greeks understood that fact and applied the golden ratio on their constructions. Consequently, in order for architects to make a beautiful building they have to use the golden ratio.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

2. A Circle is a Circle

Math Topic: Geometry, History of Mathematics

Age Group: 9-13

Knowledge Background: Circle, Diameter, ratio of a circle's circumference to its diameter, basic knowledge of mathematics, π, concept-term relations

Knowledge Acquired: Chord of a Circle, history of mathematics, the main developments of Pi through the centuries, the surprisingly early existence of advanced mathematics

Skills Acquired:

The preparation and presentation required for this MATHFactor aids the Comprehension of pupils with respect to:

- understanding historical facts
- discovering historical facts
- analyzing historical facts in reading materials

Initially, the student has to collect a lot of information and carefully select which examples are appropriate and easy to understand for non-mathematicians. Finally, he/she needs to plan the presentation.

Mathematical Modeling Skills - a real life problem is presented as a mathematical problem (e.g. King Salomon's round water basin). The historical mathematical solutions are analyzed and then related back to the real life solution. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Numerical and Symbolic Computation - needed in order to understand the different solutions used throughout history.

Visualization Skills - developed, as graphical drawing is needed in order to visualize both the mathematical solution and observation of the content.

Use and Applicability: History has shown a lot of mathematical models which can be used to solve important problems in daily life. It can be seen that the use of creative thinking is the best.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

3. A trip to the moon

Math Topic: Mathematical algorithms, estimations

Age Group: 9-13

Knowledge Background: Unit conversion, mm, cm, m, km, Multiplication

Knowledge Acquired: Power of a number, application of the formula u=s/t

Skills Acquired:

The presentation is based on using mathematics theory to solve an imaginary problem. However, in order to start solving the problem, the student has to comprehend it first.

Mathematical Modeling Skills - the mathematical modeling theory states that a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Analytical Thinking - trying to solve the problem by using different methods, finding the time needed for a trip when you know the speed and the total distance, finding the number of steps and finding the power of a number in order to solve a problem form the basis of analytical thinking.

Applicability - needed since the student has to apply the knowledge acquired to solve the problem.

Communication – skill of presenting a mathematical idea (mathematics communication).

4. Busy as a bee – mathematics and mysteries of nature

Math Topic: Geometry

Age Group: 9-13

Knowledge Background: Basic geometry

Knowledge Acquired: Strength of different geometrical figures

Skills Acquired:

- Understand and explain geometrical figures
- Communicate real life with science and mathematics
- Reasoning and critical thinking

5. Camping

Math Topic: geometry

Age Group: 9-13

Knowledge Background: midpoint

Knowledge Acquired: Definition and Properties of perpendicular bisector, definition and properties of circumcenter, finding the center of a circle

Skills Acquired:

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it.

Mathematical Modeling - the mathematical modeling theory states that a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Analytical Thinking - there is a number of stages which enhance the development of analytical thinking skills. These include analysing and separating the problem into its constituent parts and finding the perpendicular bisector of two points. The point of intersection of two perpendicular bisectors is equidistant from the three original points, so their point of intersection gives the centre of the circle.

Visualization Skills - developed, as graphical drawing is needed in order to visualize both the mathematical solution and observation of the problem.

Use and Applicability: In various situations we often have two or three points and we need to find an ideal position for a new item or building and further support our decision with a logical proof of our conclusion. This supports the use of mathematical logic and appreciation of its application in real life problems, such as finding the right place for a bus station.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

6. Creation of Conics

Math Topic: Conics

Age Group: 14-18

Knowledge Background: Understanding basic geometrical concepts, functions and cross sections. It does not require any other specific knowledge. It is recommended but not necessary that the pupils get acquainted with conics, especially with their focal points and directrix

Knowledge Acquired: The ability to follow mathematical instructions presented in the verbal form, a better understanding of conics – their focal points and directrix and relationship between an object and its tangents – is expected. In order to obtain correct conics, precise folding is required.

Skills Acquired:

The story shows possibilities of the use of dynamic geometry in visualization and modeling of non-standard problems. It also represents a non-traditional model of conic design. It is one of few activities in which pupils create a curve in a way other than drawing.

Problem Solving - stimulating is the part that can be done in the form of inquiry-based learning, where the pupil has to consider how an object is created by folding a piece of paper and further understand what the relationship between individual folds and the conic is. Pupils work intuitively with concepts that go substantially beyond the level of secondary mathematics.

The understanding of the assignment requires the development of the pupils' **ability to mathematize the situation described in words** and **to work precisely.**

Fine Motor Skills - especially valuable nowadays, since they are not developed enough by the "computer generation" and some activities (e.g. precise drawing) are replaced by computers.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools. The activity contributes to the development of the pupils´ personalities by increasing their **self-confidence** and other personality traits. Moreover, it contributes to better future performance of students in the field of mathematics, as it makes the subject more popular.

7. Covering a chess board with dominoes

Math Topic: number theory

Age Group: 9-13

Knowledge Background: Odd and even numbers

Knowledge Acquired: Application of number theory, importance of mathematical proof

Skills Acquired:

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it.

Mathematical Modeling - the mathematical modeling theory states that a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Analytical Thinking - there is a number of stages which enhance the development of analytical thinking skills. These include analysing and separating the problem into its constituent parts, separating each domino to black and white and comparing them with the chessboard.

Visualization Skills - developed, as graphical drawing is needed in order to visualize the mathematical solution and observation of the problem.

Use and Applicability: In both number theory and mathematical modeling, the solutions provide a logical proof of the conclusion. This supports the use of mathematical logic and appreciation of its application in real life problems, such as covering an area with tiles.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

8. Curry's Triangle

Math Topic: Geometry

Age Group: 14-18

Knowledge Background: Trigonometry: tangent formula, irreducible fractions, corresponding angles

Knowledge Acquired: Critical thinking, be wary of appearances

Skills Acquired:

Analytical Thinking - needed throughout the different steps of the demonstration.

Visualization Skills - developed, as graphical drawing helps to visualize both the mathematical solution and observation of the problem.

Kinesthesic and Spatial Skills - developed, as the student manipulates wooden elements on the plans of the two boards and arranges the shapes together.

Use and Applicability: This presentation is a good way to reinvest and/or deepen geometrical basic notions, via a magic trick. Other ways of finding the solution are possible and other geometrical notions could be used in the presentation.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

9. Find the mistake

Math Topic: Geometry

Age Group: 13-18

Knowledge Background: Circle, Diameter, Centre, Circumscribed circle, cyclic quadrilateral, perpendicular line, angle at the circumference

Knowledge Acquired: Properties of cyclic quadrilaterals, properties of circles, Thales Theorem

Skills Acquired:

Problem Solving - the preparation and presentation requires the development of the pupils' problem-solving skills.

In addition, understanding the assignment requires the development of the pupils' ability to mathematize the situation described in words and to visualize the situation.

Subsequently, looking for the mistake requires **activation of knowledge for the mathematical situation** from the relevant domain. Here, any of the facts known to pupils can be applied in a new situation or pupils can use the exact drawing.

The knowledge of 2D geometrical properties is also applied here **in a non-traditional way** which increases the motivational aspect of the problem dealt with.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

10. If you want to cross the street

Math Topic: Geometry

Age Group: 9-13

Knowledge Background: Elementary triangle theory, the law of sines

Knowledge Acquired: Deepening the understanding of the application of the law of triangles

Skills Acquired:

Critical Thinking - this presentation could be used to show the importance of the proof in mathematics, developing in this way the pupils' critical thinking skills.

Visualization Skills - developed, as graphical drawing is needed in order to visualize both the mathematical solution and observation of the problem.

Use and Applicability: Firstly, the students interact with each other and with their families in order to decide how they can cross the street. This interaction helps in understanding the real life vocabulary and provides a conclusion with respect to important real life situations.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

11. Logarithm, i.e. arithmetic locus...

Math Topic: logarithm, loci

Age Group: 14-18

Knowledge Background: addition, multiplication, numbers, exponentiation

Knowledge Acquired: Putting logarithms into practice, discovering logarithms, using logarithms in mathematical calculation, logarithmic calculation tables

Skills Acquired:

This presentation envisages the use of mathematical concepts in real life, particularly in transatlantic navigation when both the lives of the people on board and the reputation of the companies depend on the accuracy of the calculation.

In order to understand the problem which emerged centuries ago, the students need to grasp its true power, have an analytical approach, try to visualize and match the new issues with the already acquired ones, as well as combine and assimilate them. The ultimate target is the awareness of the fact that the newly learned item is a wonder of mathematics through its miraculous capacity of turning the multiplication into addition.

Mathematical Modeling - the mathematical modeling theory states that a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Communication – skill of presenting a mathematical idea (mathematics communication).

12. The ideal number of weights

Math Topic: Number Theory (numeral systems)

Age Group: 14-18

Knowledge Background: divisibility with remainder, powers of numbers, geometric progression, formula for the sum of a geometric progression

Knowledge Acquired: existence of numeral systems which are different of the 10 base one; how to represent natural numbers in 3-base numeral system

Skills Acquired:

Analytical Thinking – analysing the mathematical problem into its constituent parts and finding the remainder in division by 3 provide the necessary evidence for the development of analytical thinking skills.

Logical Reasoning – different ways of measuring and weighing.

Mathematical Modeling – a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution.

Problem Solving – in order to start solving the problem, one should firstly comprehend the conditions and plan the solution.

Communication – skill of presenting a mathematical idea (mathematics communication).

13. The Little Red Riding Hood and Diophantine Equations of First Order

Math Topic: Diophantine equations

Age Group: 9-13

Knowledge Background: Linear Diophantine Equations with two variables, common divisor, prime number, co-prime numbers

Knowledge Acquired: ability of modeling, how to check the existence of a solution of a linear Diophantine Equation with two variables

Skills Acquired:

Analytical Thinking – analysing the mathematical problem into its constituent parts, finding the common divisors or checking whether two numbers are co-prime provide the necessary evidence for the development of analytical thinking skills.

Mathematical Modeling – a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution.

Problem-Solving – in order to start solving the problem, one should firstly comprehend the conditions and plan the solution.

Communication – skill of presenting a mathematical idea (mathematics communication).

14. The invariant property

Math Topic: Invariants

Age Group: 9-13

Knowledge Background: counting, addition, subtraction of integers, even and odd integers

Knowledge Acquired: the definition of invariant, ability of detecting invariant property

Skills Acquired:

Analytical Thinking – analysing the parity of integers and checking whether an integer is even or odd provide the necessary evidence for the development of analytical thinking skills.

Mathematical Modeling – a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution.

Problem Solving – in order to start solving the problem, one should firstly comprehend the conditions and plan the solution.

Communication – skill of presenting a mathematical idea (mathematics communication).

15. Egyptian Fractions

Math Topic: Ordinary fractions

Age Group: 9-13

Knowledge Background: ordinary fraction, summation of ordinary fractions with one and the same denominator, divisor, and proper divisor.

Knowledge Acquired: definition of Egyptian fraction, ability of modeling, perfect number, how to check that a number is perfect, historical facts.

Skills Acquired:

Analytical Thinking – analysing the mathematical problem into its constituent parts and finding the divisors of an integer provide the necessary evidence for the development of analytical thinking skills.

Mathematical Modeling – a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution.

Problem Solving – in order to start solving the problem, one should firstly comprehend the conditions and plan the solution.

Communication – skill of presenting a mathematical idea (mathematics communication).

16. How did Eratosthenes manage to calculate the circumference of the Earth 200 years BC?

Math Topic: Geometry

Age Group: 14 -18

Knowledge Background: Circle, sphere, angle

Knowledge Acquired: Calculus of circumference, ratios, size conversion

Skills Acquired:

Analytical Thinking and Problem Solving - the main skills acquired through this presentation, as it involves a step-by-step methodology for solving a problem that includes its understanding and then gathering and combining information in order to reach a conclusion /solution.

Use and Applicability - This is highlighted here as well, as the story is about a practical problem with a profound application in geography and geodesy.

Visualisation Skills - boosted because of the shape which is necessary in order to fully understand the problem.

Mathematical Modeling - the Earth and the Sun system are represented with the help of a sphere and flashlight. The Earth and the Sunrays are subsequently represented with the help of a hoop and wooden sticks.

The way this script is presented involves gathering information and identifying key issues related to it. Consequently, it boosts **analytical thinking** and **problem-solving skills**. It also places calculations in a frame of use and application, as it highlights the connection of Mathematics and Physics. By presenting this script, students will also gain **mathematics communication** skills.

17. Hidden Paths and Patterns

Math Topic: Algebra

Age Group: 14- 18

Knowledge Background: Mathematical operations

Knowledge Acquired: Modeling tricks, pattern spotting, pair up method, reverse doubling method

Skills Acquired:

Problem Solving and Analytical Thinking - this script starts and ends with the understanding of a problem and then the different approach we can take to solve it. As a result, it helps the students build their problem- solving and analytical skills.

Numerical Computation and Modeling - it has elements that boost numerical computation skills and it is all based on modeling skills, as it reveals two of the most useful techniques for finding patterns and modeling problems.

Finally, it matches modeling to real life problems that develop the **use and application** of mathematics skills, while the presentation of the script helps students present their ideas and understand how **mathematics communication** works.

18. How does Santa make it?

Math Topic: Arithmetic

Age Group: 9 - 13

Knowledge Background: Mathematical operations, division, percentages, time difference, average

Knowledge Acquired: Calculus of speed, hour to seconds and backward conversion, calculus in general

Skills Acquired:

Problem Solving and Analytical Thinking - the way this script is presented involves gathering information and identifying key issues related to it. In this way, it boosts analytical thinking and problem-solving skills.

It also places calculations in a frame of **use and application**, as it highlights the connection of Mathematics and Physics. By presenting this script, students will also gain **mathematics communication** skills.

19. Lucky bet

Math Topic: Algebra – Probability Theory

Age Group: 14- 18

Knowledge Background: Mathematical operations, percentages

Knowledge Acquired: Ratios and probabilities

Skills Acquired:

Problem Solving and Analytical Thinking - the history of Chevalier de Mere's problem is one that develops both the analytical thinking and the problem solving skills of the students, as they have to understand the problems and then gather all the necessary information, analyse it and reach a conclusion.

It is also a matter of **numerical computation**, as it is needed in order to calculate the odds. This is highly connected with **use and application** in our everyday life, as the whole section of probability theory is. The way it is presented takes advantage of an interesting bit of mathematical history, required to carry out a **mathematics communication** talk.

20. The sound of music

Math Topic: Algebra

Age Group: 9-13

Knowledge Background: Frequency

Knowledge Acquired: Ratio, octave, musical patterns

Skills Acquired:

This script brings together information drawn from different fields of Maths and Physics in order to explain the connection between Music and Maths. The way this is done develops the **analytical skills** of the students. Furthermore, it helps the **comprehension** of a topic and its vivid examples and metaphors, such as connecting the size of the string with a ratio, help with the visualization of the topic. Finally, it uses narrative for **mathematics communication.**

21. Where is another possibility?

Math Topic: Proof, logic, congruence of triangles

Age Group: 14-18

Knowledge Background: Basic geometrical notions, polygons in 2D

Knowledge Acquired: Application of properties of triangles and perpendicular bisectors

Skills Acquired:

Problem Solving and Analytical Thinking - the story significantly develops analytical thinking and the ability to solve problems. Students must seek different views of the current problem, model a variety of situations and critically evaluate these models.

Visualization of the models has a great importance.

The story also develops comprehension of the concept of congruence of triangles.

The Communication skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

22. Irrationality of square root of 2

Math Topic: Irrational numbers

Age Group: 14-18

Knowledge Background: Pythagoras' theorem, rational numbers, irreducible fractions, remarkable identities

Knowledge Acquired: Irrational numbers (e.g square root of 2) demonstrate an intermediate property, i.e. if the square of an integer is an even number, its number is an even number as well, Reasoning/demonstration of ad absurdum, History/Philosophy of mathematics

Skills Acquired:

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Analytical Thinking - needed throughout the different steps of the demonstration.

Visualization Skills – developed, as graphical drawing helps to visualize both the mathematical solution and the observation of the problem.

Use and Applicability: This topic provides an easy way to demonstrate the ad absurdum, perhaps for the first time in the students' curriculum. By this the students can realise how important this discovery can be.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

23. The Monty Hall Show

Math Topic: Probabilities

Age Group: 14-18

Knowledge Background: basic logic

Knowledge Acquired: Basic probabilities; this presentation can also lead to the discovery/introduction of probability tree diagrams

Skills Acquired:

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Visualization Skills - developed, as graphical drawing helps to visualize both the mathematical solution and the observation of the problem.

Use and Applicability: In various situations where probabilities are needed. This presentation exhibits in a humorous way that, although our instinct can lead us the wrong way, probabilities help us to find the right way.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

24. Playing Tetris

Math Topic: Playing Tetris

Age Group: 9-18

Knowledge Background: No background needed

Knowledge Acquired: Basic knowledge in number theory

Skills Acquired:

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Analytical Thinking - analysing and separating the problem into its constituent parts through colouring the playing field and each piece in two colors (in order to solve the problem) provide evidence of the development of analytical thinking skills.

Visualization Skills - developed through the rotation and movement of the pieces left and right. This is needed in order to explain the game, while colouring the blocks is needed in order to visualize both the mathematical solution and the observation of the problem.

Use and Applicability: We can see how odd and even number knowledge can be applied. This supports the use of mathematical logic and the appreciation of its application to real life problems, like this problem which has evolved from a game.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

25. To tell a lie or to tell the truth? That is the question!

Math Topic: The formulation of logical statements

Age Group: 9-13

Knowledge Background: None

Knowledge Acquired: Logical statements, logical reasoning, and logical value of true and false statements

Skills Acquired:

Analytical Thinking - analysing and separating the problem into its constituent parts and taking cases in order to come to a final idea provide evidence for the development of analytical thinking skills.

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

The mathematical didactics emphasize the motivation for problem solving. A problem is placed in a fictional environment, but is subsequently translated to a mathematical problem in order to find its mathematical solution and finally translate it back to fiction.

Visualization Skills - developed as a piece of the history of mathematics.

Use and Applicability: This principle is very important for logics and some problems can be solved through this method, while in other cases it helps logical reasoning and corrects the formulation of statements.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

26. Pigeonhole Principle

Math Topic: The Pigeonhole Principle

Age Group: 14-18

Knowledge Background: Indirect proof, logical reasoning

Knowledge Acquired: Pigeonhole Principle

Skills Acquired:

Analytical Thinking - analysing and separating the problem into its constituent parts and taking cases in order to come to a final proof provide evidence for the development of analytical thinking skills.

Logical Thinking - the preparation and presentation required for this MATHFactor develops the reasoning, logical thinking, deducing and arguing of the pupils. This happens because the pupil firstly needs to comprehend the problem, plan its solution and then start solving it.

Visualization Skills - developed through the visualization of the pigeons going into the pigeonholes, and used in order to visualize both the mathematical solution and the observation of the problem.

Use and Applicability: This principle is very important for number theory, graph theory and in solving many problems.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

27. The Tower of Hanoi

Math Topic: The mathematical induction for the number of steps to solve the Tower of Hanoi

Age Group: 9-13

Knowledge Background: basic operations with powers

Knowledge Acquired: The principle of mathematical induction

Skills Acquired:

Communication - the strategy of the game is based on mathematics, modeling the problem and manual handling of the discs. The acting and the use of visual models develop the Communication skills of the pupils.

Methodology - Practical learning, explanation for a deeper understanding and modeling.

Analytical Thinking - analysing and separating the problem into its constituent parts and taking cases in order to come to a final proof provide evidence for the development of analytical thinking skills.

Problem Solving - the pupil firstly needs to comprehend the problem, plan its solution and then start solving it. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Visualization Skills – developed, as a figure and a wooden model exhibit a visualization of the Tower of Hanoi. These are used in order to visualize the mathematical solution and the follow up of the problem.

Use and Applicability: This principle is very important for number theory and problem solving.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

28. Clever squaring

Math Topic: The mathematical induction for the number of steps to solve the Tower of Hanoi

Age Group: Age 9-13

Knowledge Background: basic operations with powers

Knowledge Acquired: The “clever” formula for squaring a two - digit number

Skills Acquired:

Communication - shorter and simpler way of computation. The use of computation develops the Communication skills of the pupils.

Methodology - Practical learning, explanation for a deeper understanding and modeling. The given formula leads to more effective computational skills.

Analytical Thinking - analysing and separating the problem into its constituent parts and taking cases in order to come to a final formula which is easy to memorize provide evidence for the development of self-confidence.

Analysis

The preparation and presentation required for this MATHFactor script develops the strategy of application of the symbolical and algebraic skills of the pupils.

According to mathematical didactics, the smart computational methods (which can be easily memorized) help the acquisition of strong and reliable computational skills. The students are always open to apply a simple way instead of a more complicated one.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

29. The Circle and the others

Math Topic: Geometry

Age Group: 9-13

Knowledge Background: Circle, Straight line, quadrilateral, polygon

Knowledge Acquired: Chord of a circle, properties of the diameter, properties of tangent and properties of regular polygons

Skills Acquired:

Communication - the preparation and presentation required for this MATHFactor develops the Communication skills of the pupils. This happens because in order to present these properties the student has to comprehend the circle.

Analytical Thinking - the analysis and separation of the properties in different parts also requires analytical thinking skills.

Visualization Skills - developed through the student touching the circle on the table to show the tangent. By touching the circle on the table in a particular way, the table edge becomes a chord and the diameter of the circle.

Use and Applicability: In geometry to introduce math students to simple properties of the circle.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

30. The loneliness of the top

Math Topic: Number Theory

Age Group: 9-13

Knowledge Background: Even numbers

Knowledge Acquired: Properties of the prime numbers, 2 is the only even prime, every number can be written as a multiplication of primes in a unique way, Historical Facts about Prime numbers, How did Eratosthenes try to find the primes?, How famous mathematicians tried to find a Prime number generator?, Euclid's proof about primes

Skills Acquired:

Organizing - the preparation and presentation required for this MATHFactor develops the organizing skills of the pupils. This is supported by the fact that in order to make the presentation the student has to comprehend the mathematics behind it and to try to plan the presentation.

Analytical Thinking - analysing and separating the history into its constituent parts that connect very nicely with one another provide the necessary evidence for the development of analytical thinking skills.

The **Communication Skills** of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

31. The Pigeonhole Principle

Math Topic: The pigeonhole Principle

Age Group: 9-13

Knowledge Background: None

Knowledge Acquired: Pigeonhole Principle

Skills Acquired:

Analytical Thinking - analysing and separating the problem into its constituent parts and taking cases in order to come to a final proof provide evidence for the development of analytical thinking skills.

Problem Solving - the preparation and presentation required for this MATHFactor develops the problem-solving skills of the pupils. This happens because the pupil firstly needs to comprehend the problem, plan its solution and then start solving it.

Mathematical Modeling - the mathematical modeling theory states that a person firstly needs to translate a real life problem into a mathematical problem, then he/she needs to find the mathematical solution and finally translate it back to the real life solution. Since all these stages are implemented, mathematical modeling skills acquisition is supported.

Visualization Skills - developed as a visualization of the pigeons going into the pigeonholes is used in order to visualize the mathematical solution and observation of the problem.

Use and Applicability: The principle is very important for number theory and many problems can be solved with the use of this principle.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

32. The story of the ladybirds

Math Topic: Algebra

Age Group: 9-13

Knowledge Background: The theory of numbers

Knowledge Acquired: Number divisibility criteria, prime numbers

Skills Acquired:

Presentation is based on the use of mathematical theories in order to solve imaginary problems.

To come up with the solution, the student must be endowed with comprehension abilities. The theory of mathematical modeling is transferred to imaginary problems and solution can be found only if certain mathematical criteria are well known.

To solve the problem, all mathematical divisibility criteria must be familiar and all members complying with these criteria, in different stages, must be eliminated. The remaining ones are to be taught as special numbers, prime numbers, both based on the **analytical thinking** and the **visualizing capacity** of the student.

By using this story, important mathematical concepts are put into practice, useful for everyday life and for developing the solving capacity in the future.

33. Where there is an X...there pops in 0, too!

Math Topic: Probabilities

Age Group: 9-13

Knowledge Background: Basic probabilities

Knowledge Acquired: Play games using math knowledge

Skills Acquired:

Problem Solving - The preparation and presentation required for this MATHFactor develops the problem-solving skills of the pupils. It is easy to understand that the preparation and promotion required develops probabilistic thinking and symbolic comprehension for students. In this respect, students learn how to play to win.

Communication - collaboration is a key component in the game development activity, and students collaborate effectively in order to create challenging games, hence developing their communication skills.

Students recognize and solve problems, develop and apply strategies based on ways others have used in order to present or solve problems.

Visualization Skills - developed, as graphical drawing is needed in order to visualize the mathematical solution and observation of the problem.

Use and Applicability: In various situations where probabilities are needed, students gather, analyse and apply information and ideas, discover and evaluate patterns and relationships in information, ideas, and structures, as well as applying acquired information and skills to different contexts as students, workers, citizens, and consumers.

The friendliness of Tic-tac-toe games makes them ideal as a pedagogical tool for teaching the concepts of good sportsmanship and the branch of artificial intelligence that deals with the searching of game trees.

34. How to generalise? What to generalise? The case of Pythagoras' theorem.

Math Topic: The application and generalisation of Pythagoras' theorem

Age Group: Age 9-13

Knowledge Background: basic form of the theorem

Knowledge Acquired: The practical application in building industry of the theorem and the generalisation for 3 and more dimensions

Skills Acquired:

Communication - application of theorems and computations. The use of computation develops the Communication skills of the pupils.

Methodology - Practical learning, explanation for a deeper understanding and modeling. The given formula leads to more effective computational skills.

Analysing and separating the problem into its constituent parts and taking cases in order to come to a final formula easy to memorize provide evidence for the development of self-confidence.

Analysis

The preparation and presentation required for this MATHFactor script develops strategy of application of the symbolical and algebraic skills of the pupils.

According to mathematical didactics, the application of computational methods (which can be easily memorized) help the acquisition of strong and reliable computational skills. The students are always open to apply a simple way in practice.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

35. How to find a rectangle when building your house? The application of Pythagoras' theorem

Math Topic: The application of Pythagoras' theorem

Age Group: Age 9-13

Knowledge Background: basic operations, square and square root, form of the theorem

Knowledge Acquired: The practical application in building the mechanism of the theorem

Skills Acquired:

Communication - application of theorems and computations. The use of computation develops the Communication skills of the pupils.

Methodology - Practical learning, explanation for a deeper understanding and modeling. The given formula leads to more effective computational skills.

Analysing and separating the problem into its constituent parts and taking cases in order to come to a final formula easy to memorize provide evidence for the development of self-confidence.

Analysis

The preparation and presentation required for this MATHFactor script develops strategy of application of the symbolical and algebraic skills of the pupils.

According to mathematical didactics, the application of computational methods (which can be easily memorized) help the acquisition of strong and reliable computational skills. The students are always open to apply a simple way in practice.

The Communication Skills of the pupils are developed through a presentation which uses the appropriate scenario, the acting and the use of visual tools.

This project has been funded with support from the European Commission.
This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.
ISBN 978-9963-713-11-0